

# Spherical Indexing of EBSD Patterns

Stuart Wright, René de Kloe

# Outline

- Hough Based Indexing
- Forward Model Based Indexing
  - Dictionary indexing
  - **Spherical indexing**
    - application results
  - Refinement
    - improved angular resolution
    - resolving pseudo-symmetry
  - Extension of Spherical Indexing



Will Lenthe



*Stuart Wright*



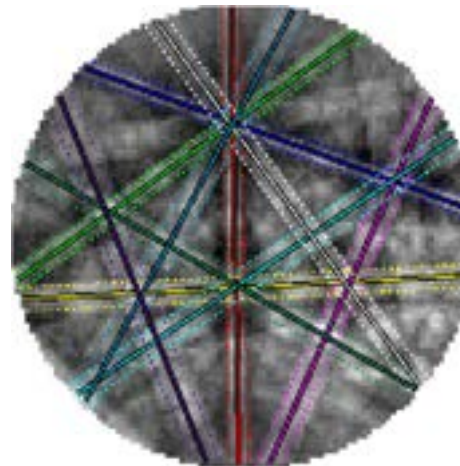
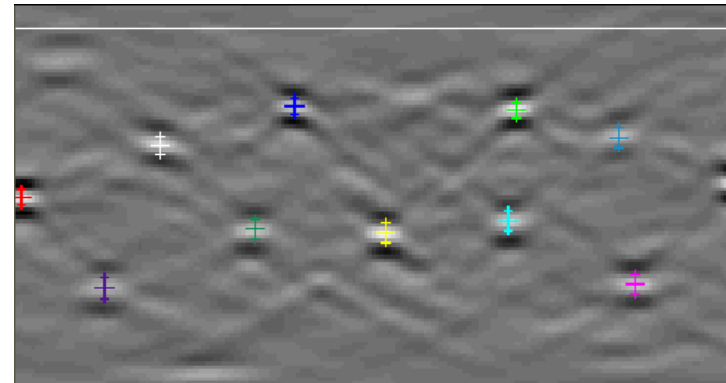
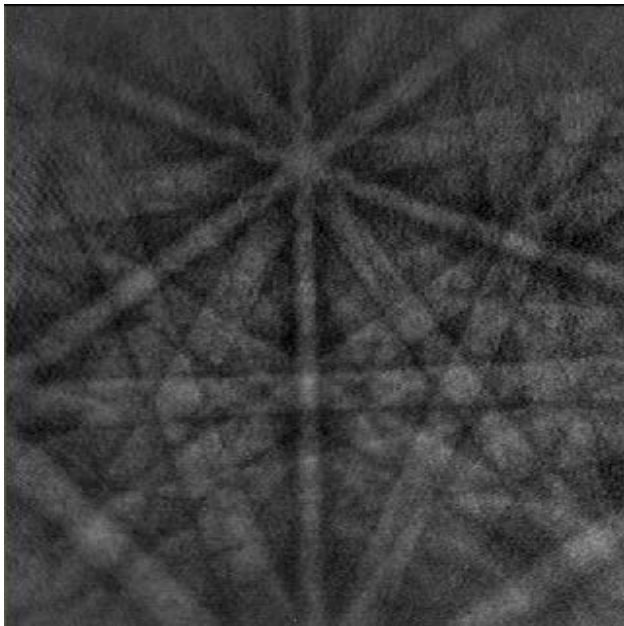
Matt Nowell



René de Kloe

# Hough transform

N. C. Krieger-Lassen, K. Conradsen, and D. Juul-Jensen (1992) "Image Processing Procedures for Analysis of Electron Back Scattering Patterns", *Scanning Microscopy*, **6**, 115-121.

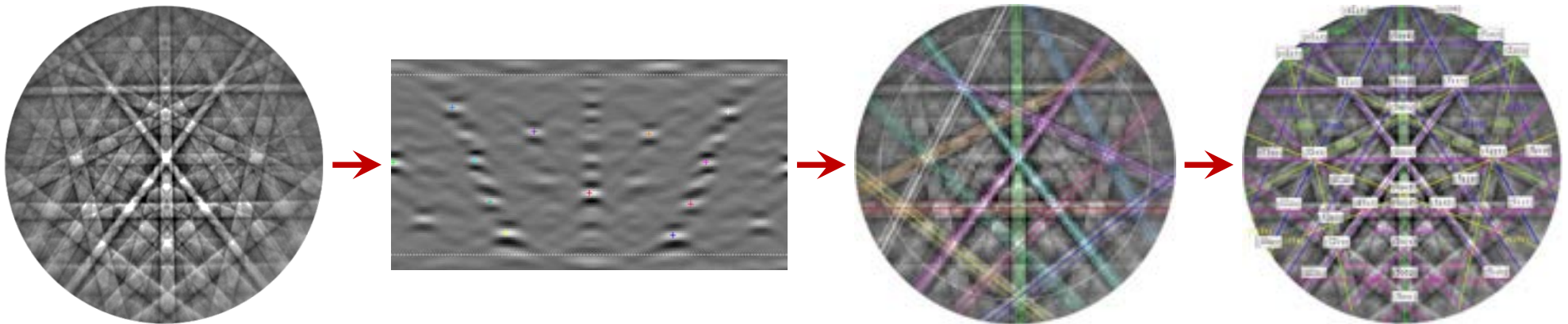


"Advances in Automatic EBSP Single Orientation Measurements"  
K. Kunze, S.I. Wright, B.L. Adams and D.J. Dingley. *Textures and Microstructures* **20**, 41-54 (1993).

Presented at *Microscale Texture of Materials Symposium, ASM/TMS Meeting*, Cincinnati, Ohio, October 1991.

# Hough transform and triplet indexing

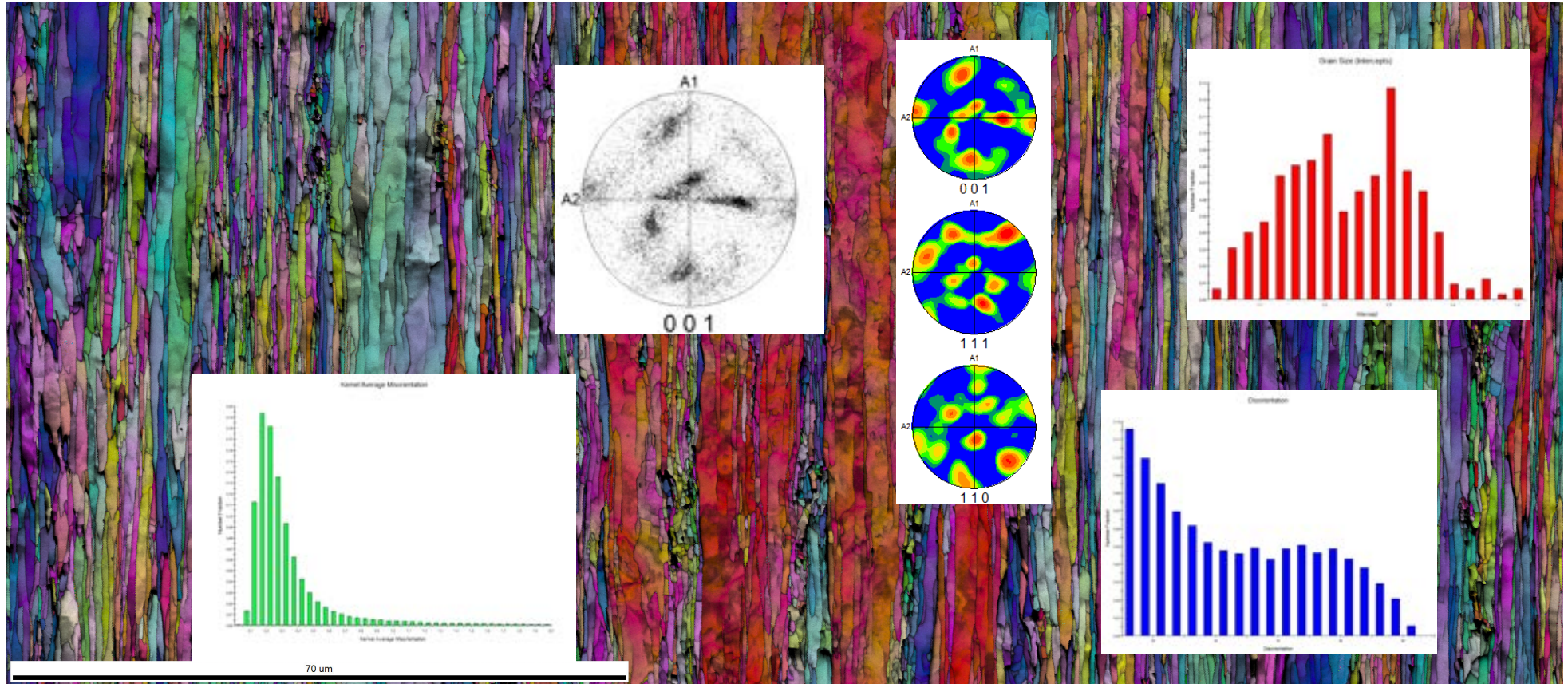
- Hough-transform based indexing method
- Collect a pattern → detect the bands to “reconstruct” the pattern → match the interplanar angles against an (ideal) reference model → find a solution



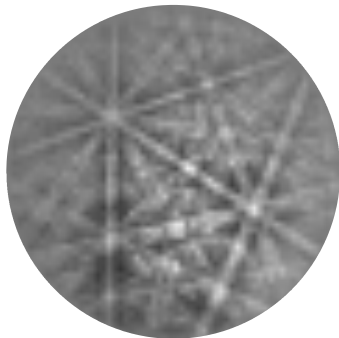
- Works very well but requires:
  - Consistent band detection
  - Detecting enough bands
  - Accurate band direction determination
  - Minimal pattern shadowing



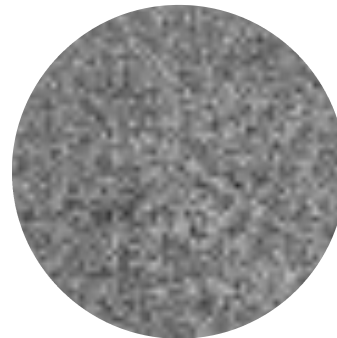
## Example → heavily deformed Mo sample with triplet indexing



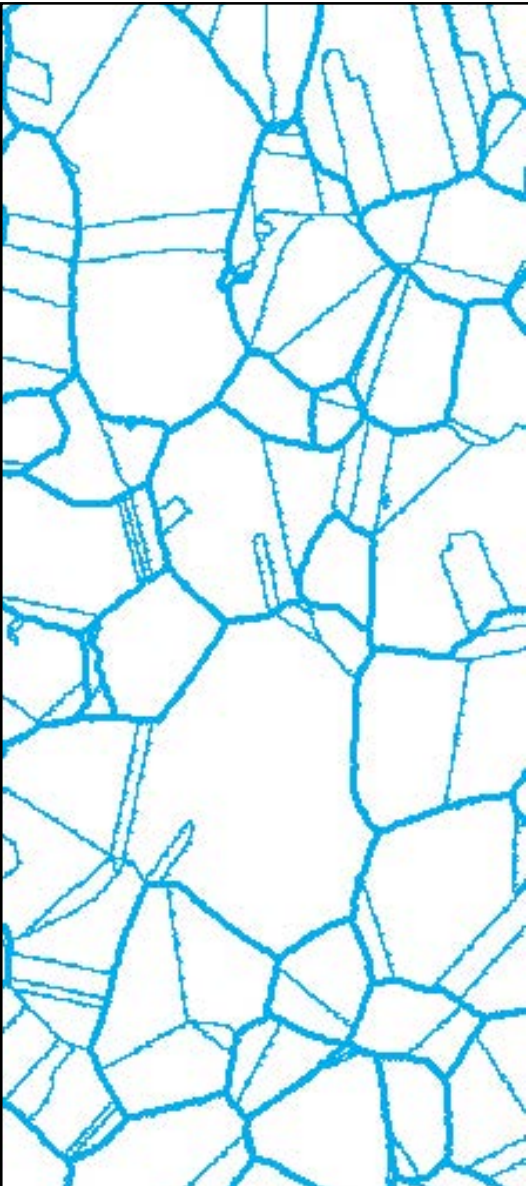
# Hough transform and triplet indexing



- Pros:
  - Fast
  - Robust in many situations
    - Particularly with N(L)PAR



- Cons:
  - Struggles with very poor patterns
  - Determining the best reflectors can be a challenge for some crystals
  - Angular resolution is  $\sim 0.1^\circ - 0.5^\circ$  primarily dependent on pattern quality
  - Difficulty resolving pseudosymmetry
  - Unable to index non-centrosymmetric crystals (band intensity profiles vs band locations)

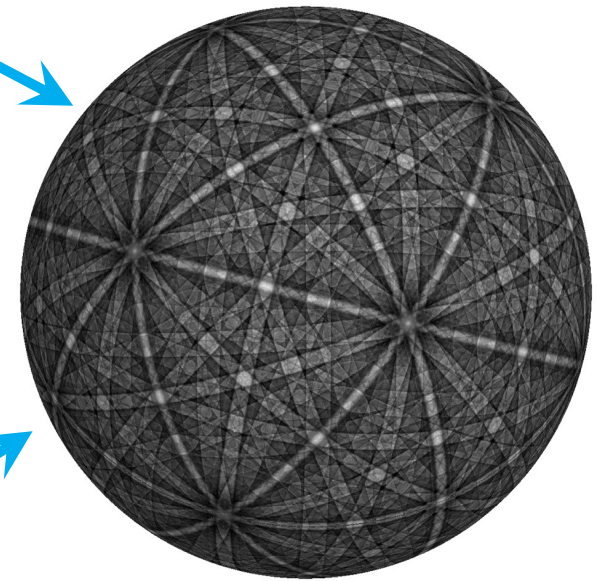
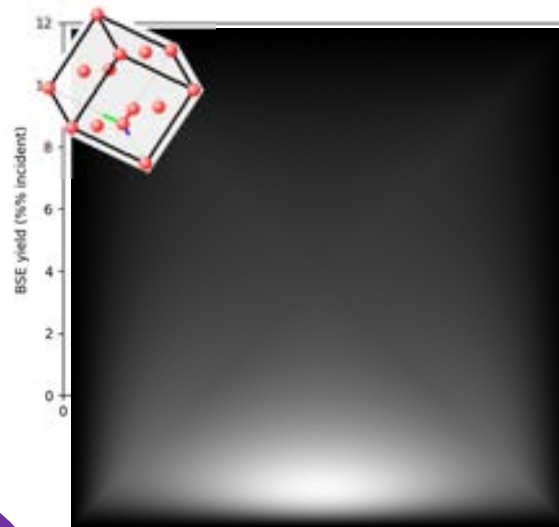
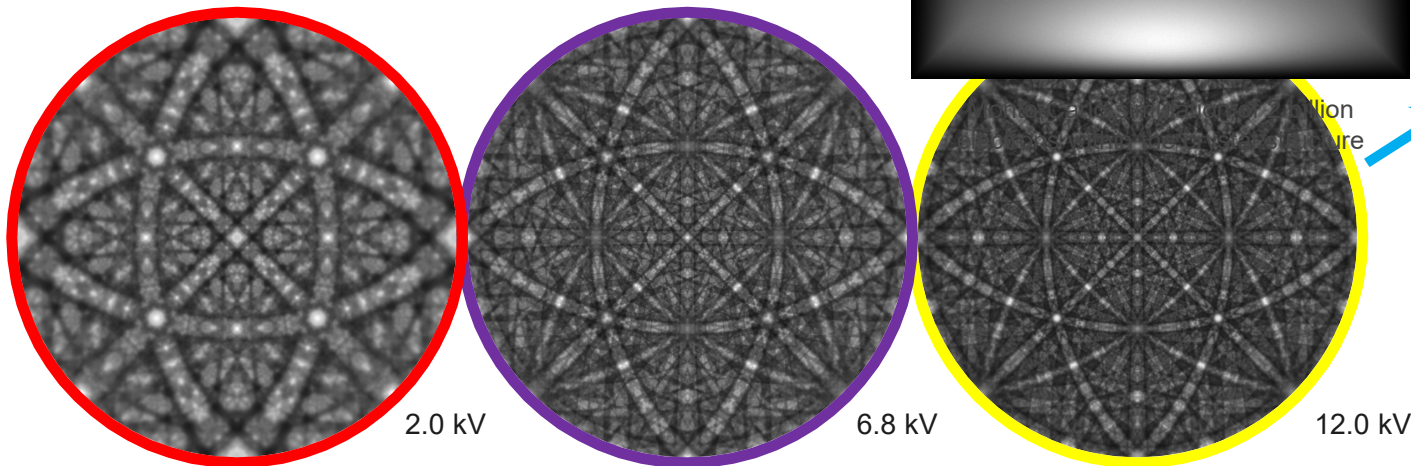


**Can we improve on Hough transform-based indexing?**



# Alternative indexing method: using dynamic pattern simulations

- Model the diffraction path of 2 billion electrons in a crystal using a Monte Carlo simulation for BSE yield
- Dynamical diffraction simulation is performed for each energy
- BSE weighted sum from all energies is combined into a single Kikuchi sphere
- Requires full knowledge of the crystal structure

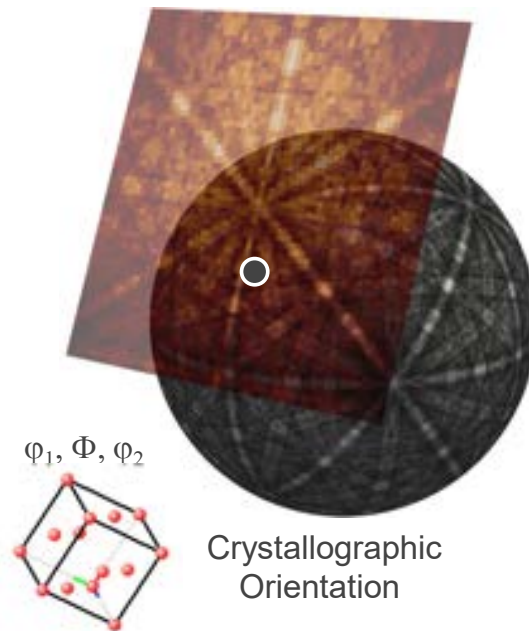
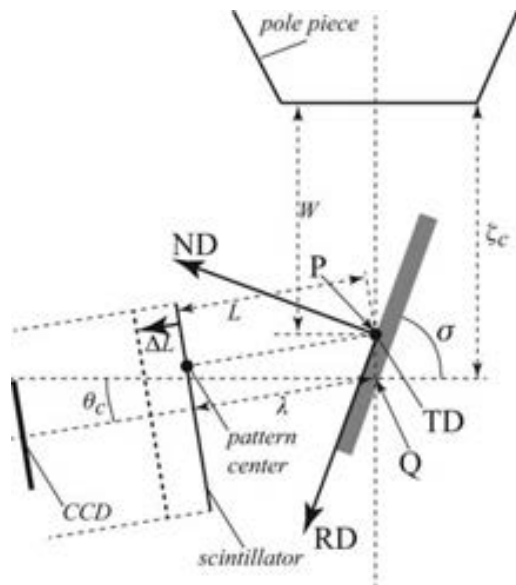


Callahan, P. G., & De Graef, M. (2013). Dynamical electron backscatter diffraction patterns. Part I: Pattern simulations. *Microscopy and Microanalysis*, 19(5), 1255-1265.

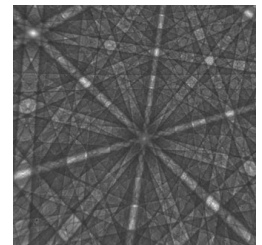
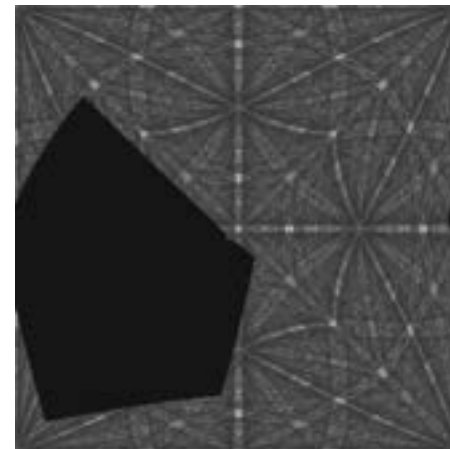
# Dynamic pattern simulation

## EBSD Pattern simulation

- Rotate Kikuchi sphere by orientation
- Project using the current calibration + crop to match the EBSD detector projection geometry



Crystallographic Orientation

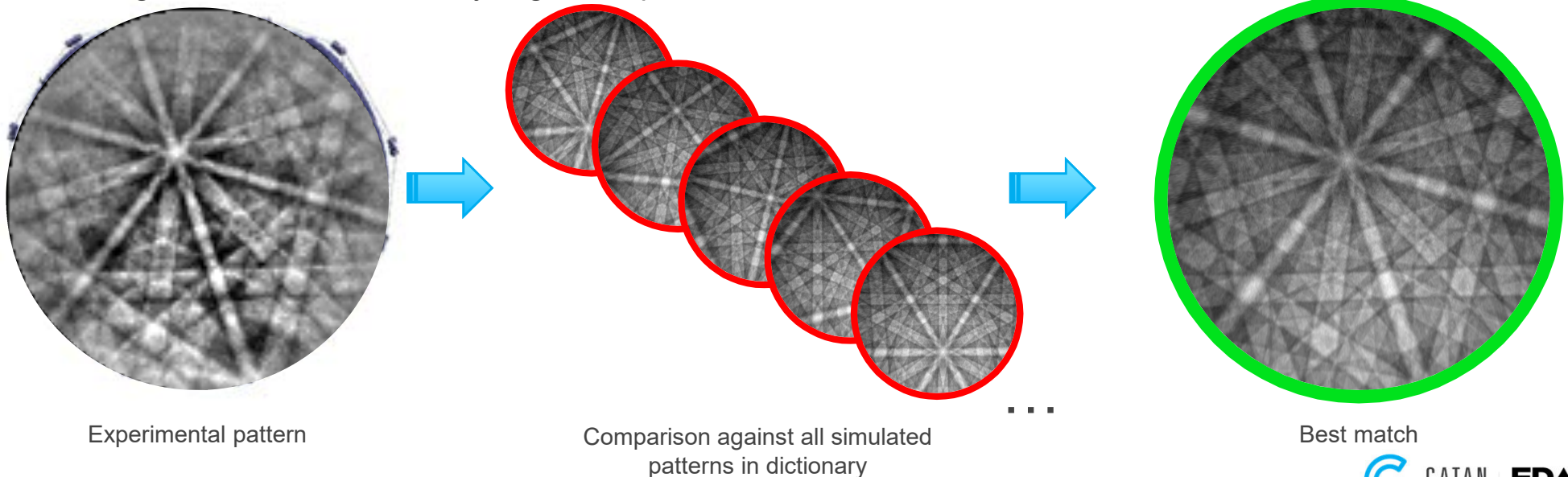


Callahan, P. G., & De Graef, M. (2013). Dynamical electron backscatter diffraction patterns. Part I: Pattern simulations. *Microscopy and Microanalysis*, 19(5), 1255-1265.



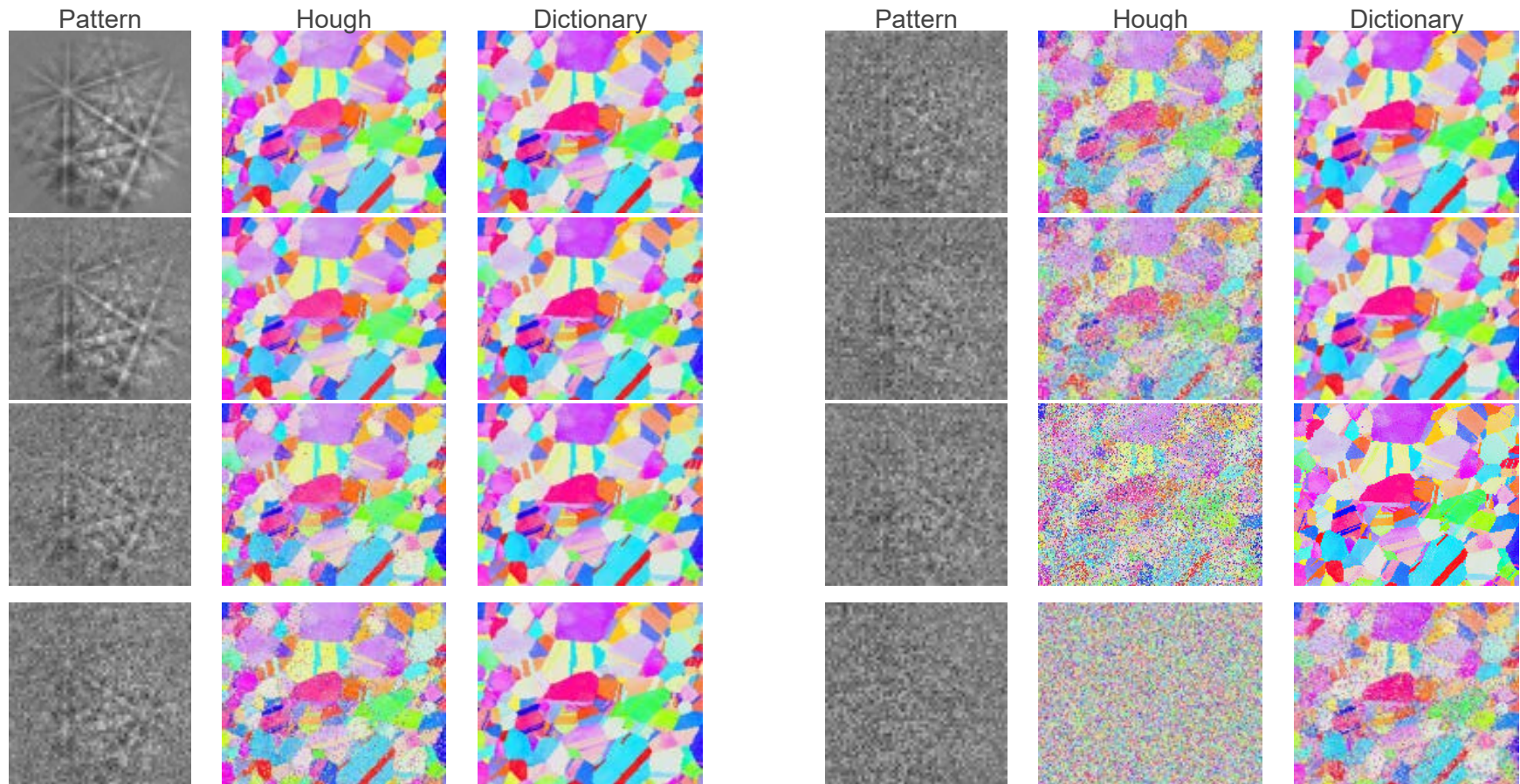
# EBSD mapping using Dictionary Indexing – procedure

- Create a library (dictionary) of all patterns of all possible orientations on a predefined grid (e.g. every  $2^\circ$ )
- Compare each pattern in a scan against all patterns in the dictionary and find the best match
- Patterns in dictionary are created to match the actual pattern projection geometry in the SEM
  - Need to create a new library for each map/WD
- Matching success is determined by largest dot product of normalized column vectors



S. Singh and M. De Graef (2016). Orientation sampling for dictionary-based diffraction pattern indexing methods.  
*Modelling and Simulation in Materials Science and Engineering*, **24**: 085013.

# Dictionary indexing robustness - Nickel with varying noise



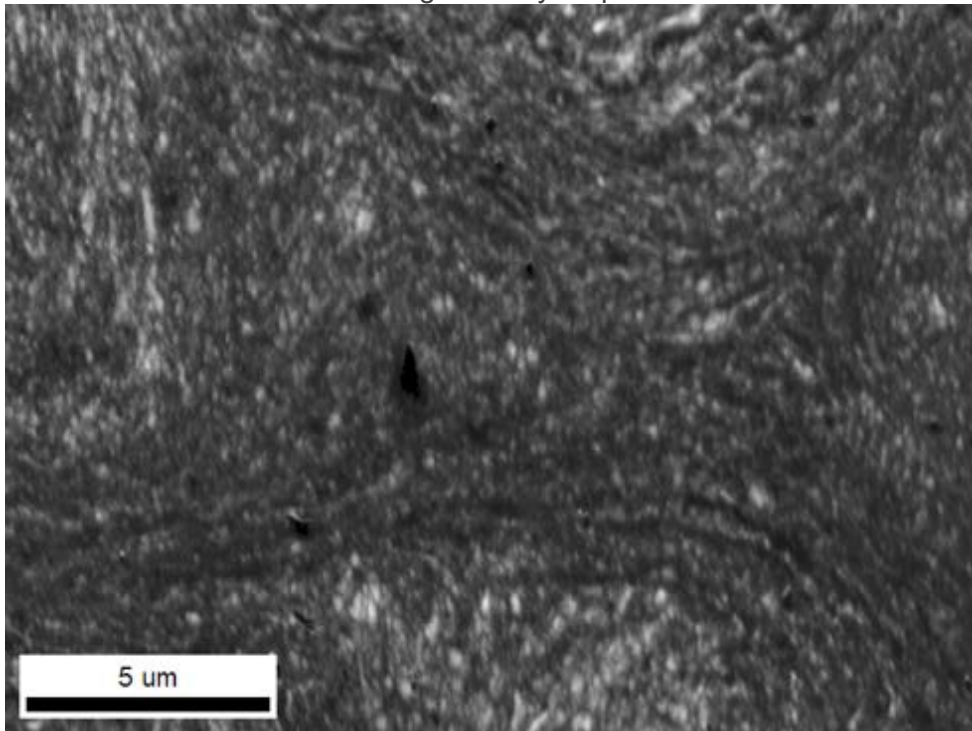
Full pattern (dictionary) matching is extremely sensitive with noisy patterns



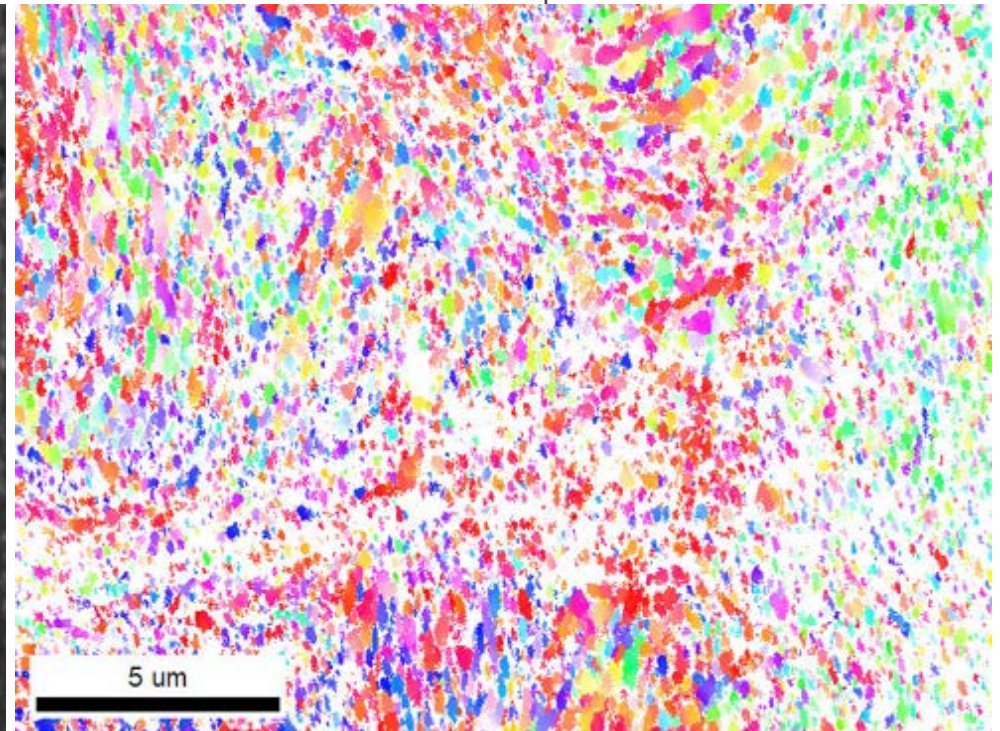
# EBSD mapping using Dictionary Indexing – example

EBSD indexing improvements on highly deformed compressed ferrite powder sample

Image Quality map



IPF map



Hough indexing, ISR: 49.8%

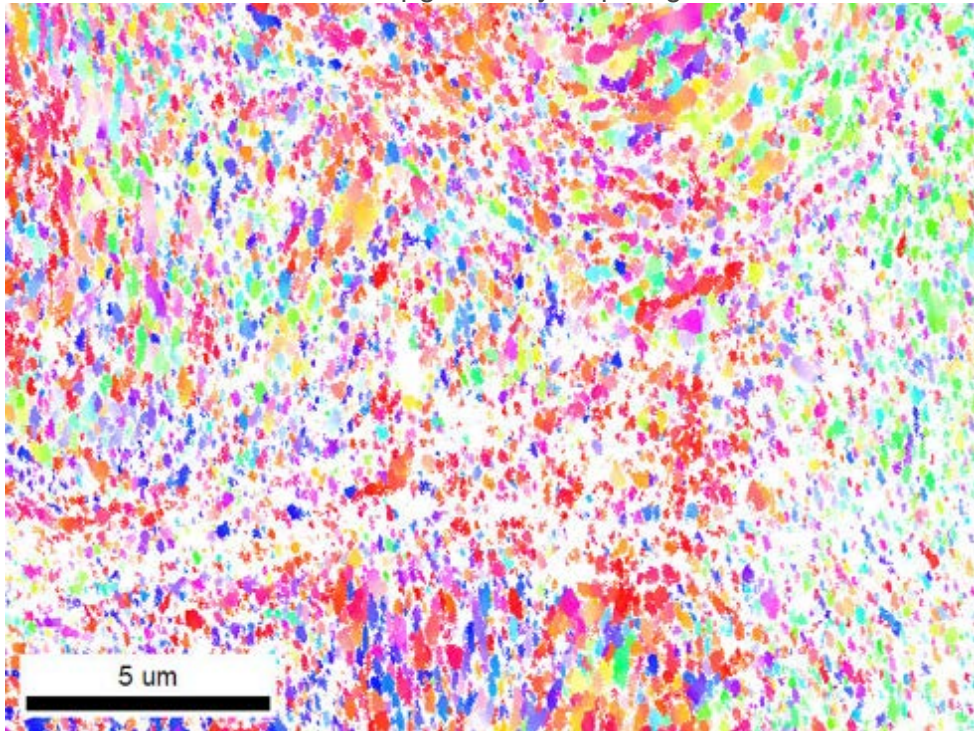
*Sample courtesy of Carsten Bonnekoh, KIT, Germany*



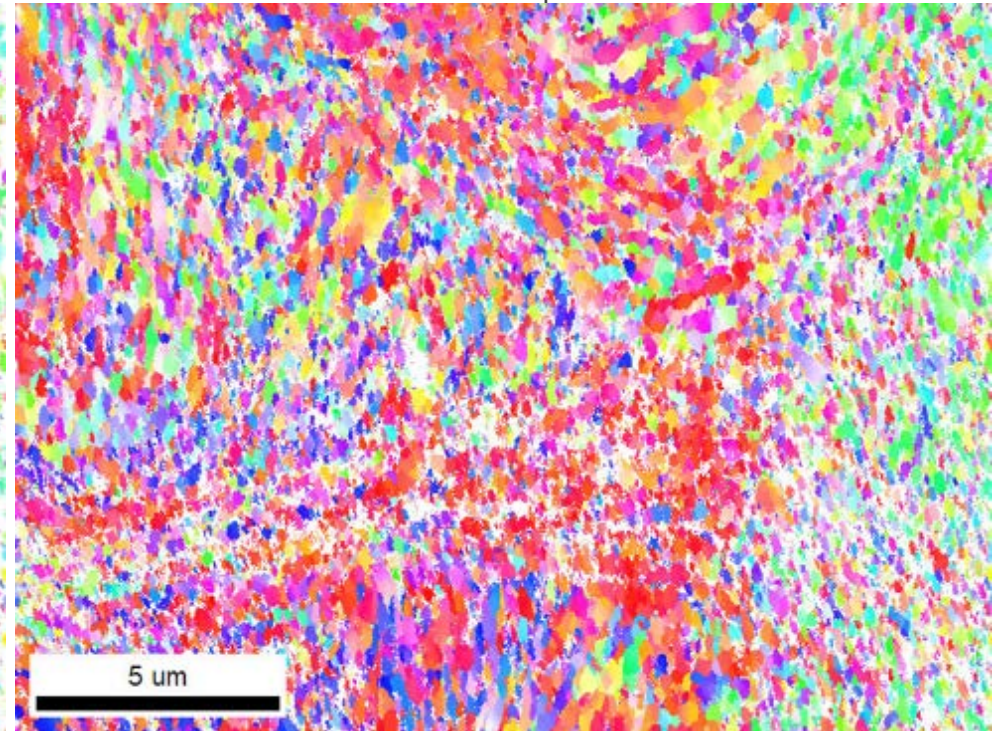
# EBSD mapping using Dictionary Indexing – example

EBSD indexing improvements on highly deformed compressed ferrite powder sample

IPF map using Dictionary indexing



IPF map



Dictionary indexing, ISR: 82.3%

Sample courtesy of Carsten Bonnekoh, KIT, Germany

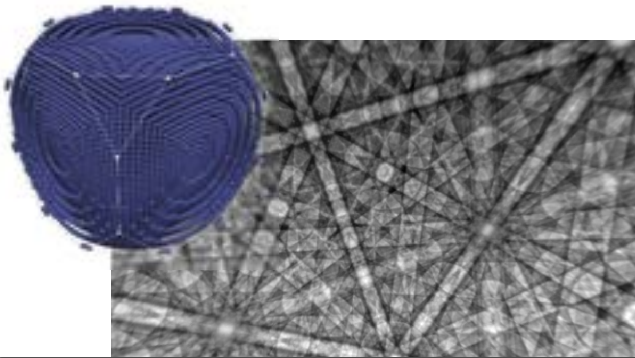
# Dictionary indexing – Pros & cons

- More robust than Hough indexing
- Significantly slower than Hough indexing
  - Large dictionaries (particularly for low symmetries)
  - Angular resolution
- Steep learning curve
  - Master pattern calculation
  - Dictionary spacing
  - Refinement parameters

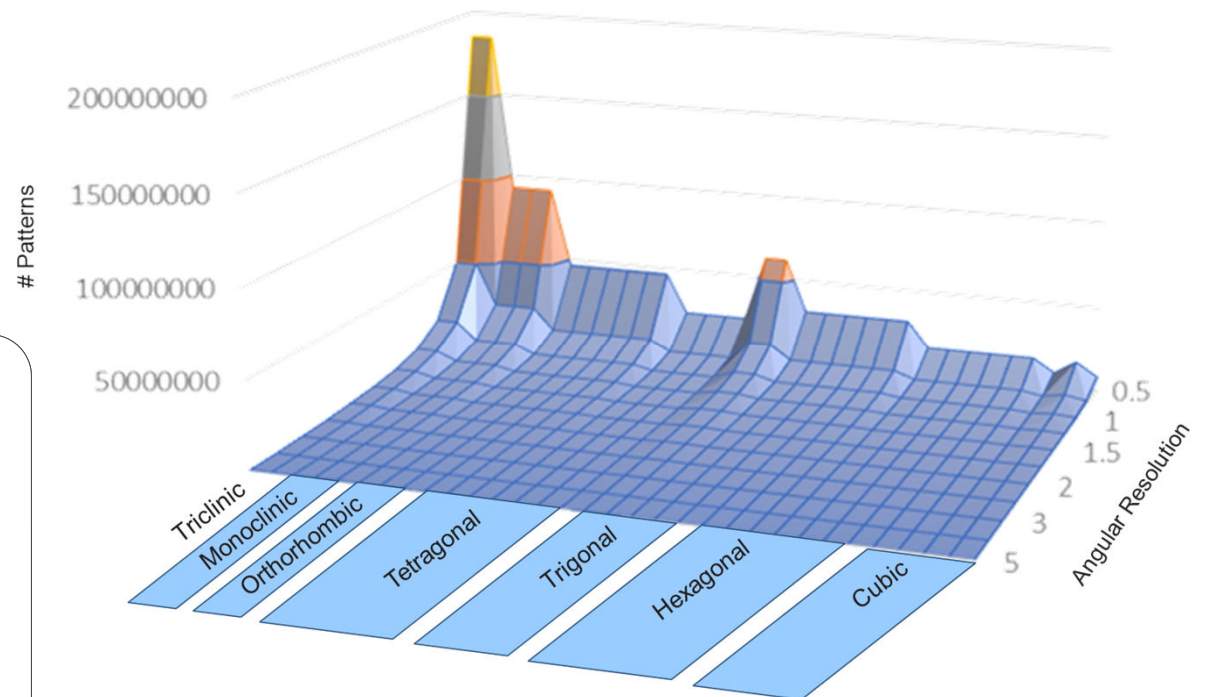
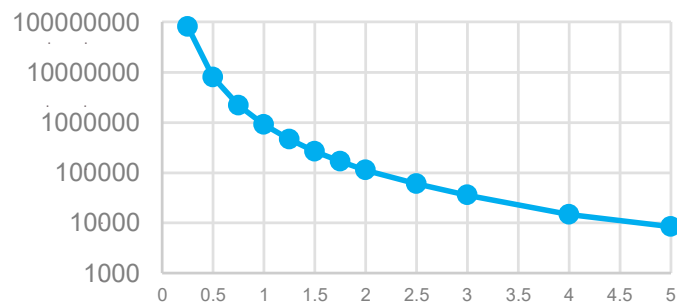


# Dictionary indexing limitation – dictionary size

- Dictionary size is related to crystal symmetry and initial grid resolution

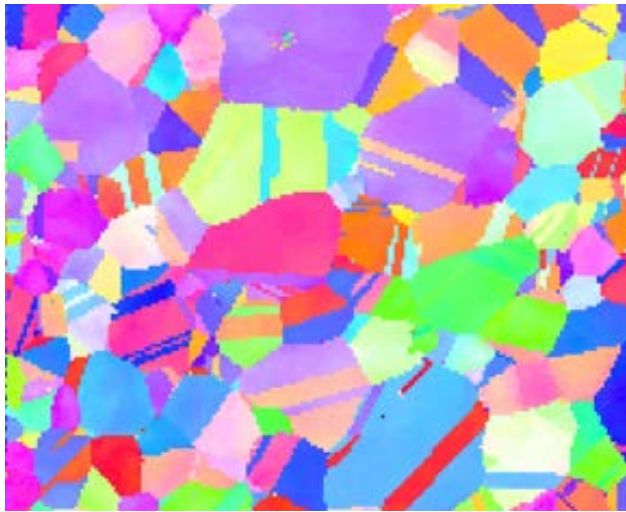


Cubic Symmetry

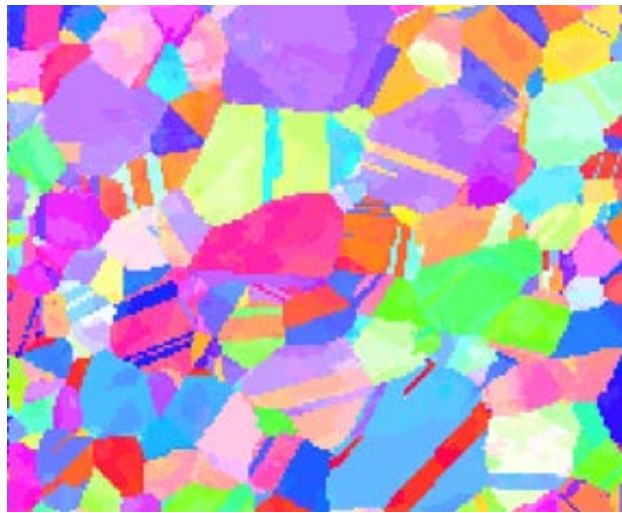


# Minimum dictionary spacing

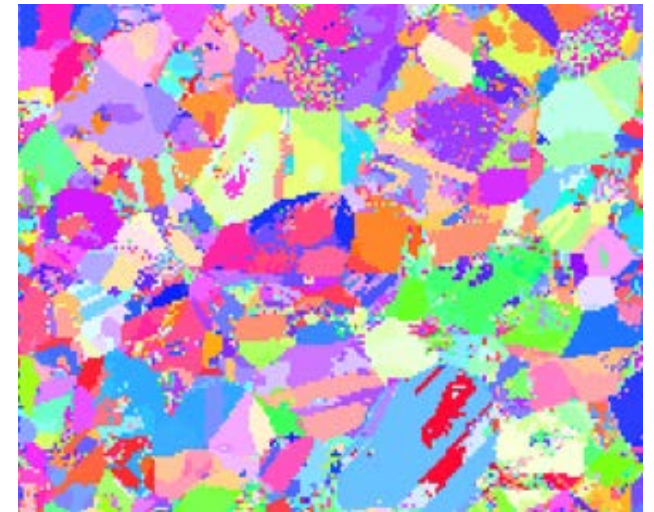
Hough-based



2.5° Dictionary



5° Dictionary



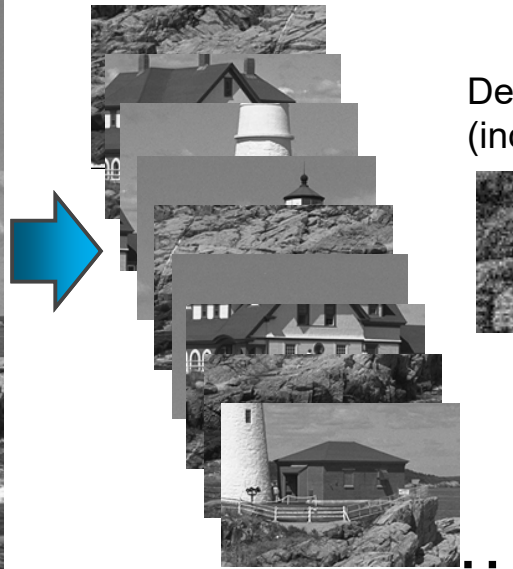
Minimum spacing is not just about angular precision but also indexing reliability

## 2D dictionary indexing

Known 'full' scene



Create a dictionary of patterns to match the detector size



Detected image  
(includes noise)



Find the best match against the dictionary to locate where the detected image falls within the full scene





## Alternative: 2D cross-correlation – Fourier transform

Known 'full' scene



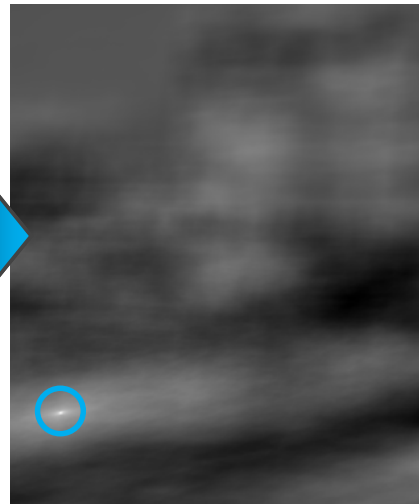
$f(x)$

Detected image  
(includes noise)



$g(x)$

Calculate cross-correlation



$$\mathcal{F}^{-1} \left\{ \overline{\mathcal{F}\{f\}} \mathcal{F}\{g\} \right\}$$

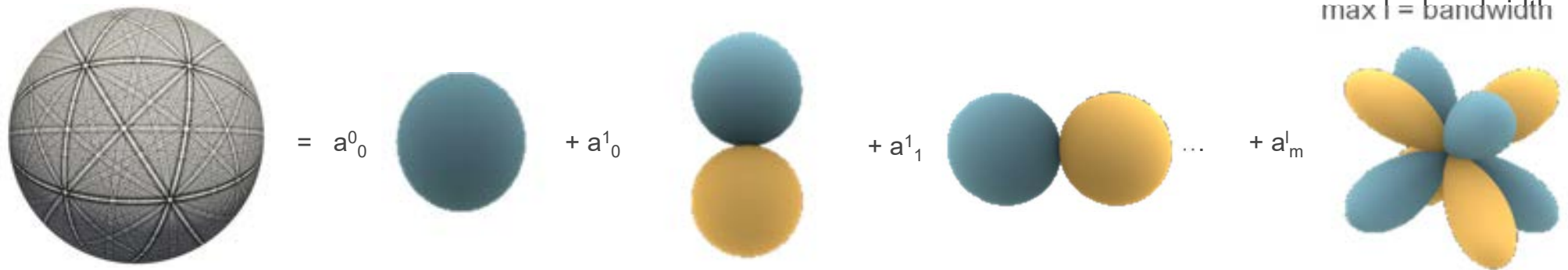
$\mathcal{F}$

Fourier transform

Use Cross-Correlation peak to  
locate detected image within scene



# Spherical harmonic transform



Fourier transform

Forward

$\mathcal{F}$

Inverse

$\mathcal{F}^{-1}$

Spherical harmonic transform

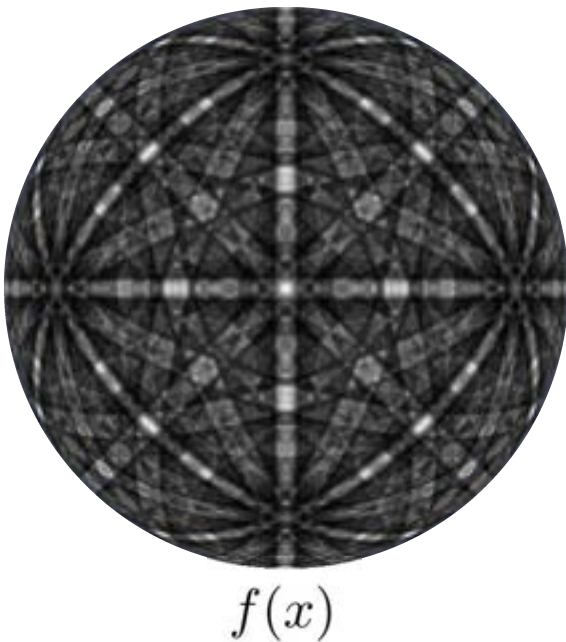
$$\hat{f}_m^l \triangleq \mathcal{S}\{f(\theta, \phi)\} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} f(\theta, \phi) \overline{Y_m^l(\theta, \phi)} \sin \theta d\phi d\theta$$

$$f(\theta, \phi) = \sum_{m=-l_{max}}^{l_{max}} \sum_{l=|m|}^{l_{max}} \hat{f}_m^l Y_m^l(\theta, \phi)$$

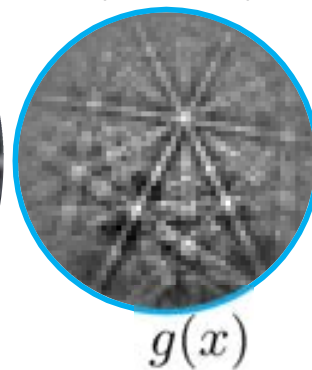


# Can we do cross-correlation on a sphere?

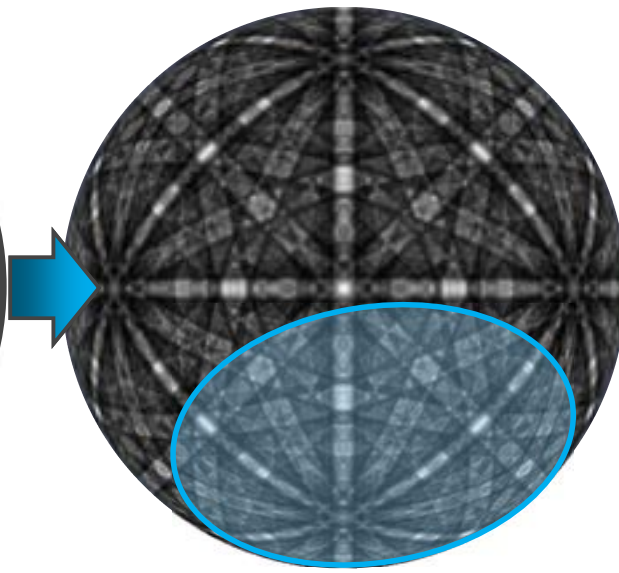
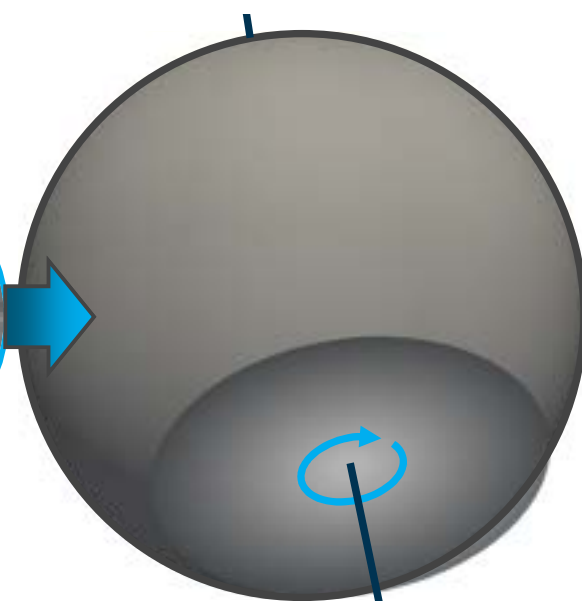
Full Scene: Simulated master  
Pattern of diffraction on a sphere



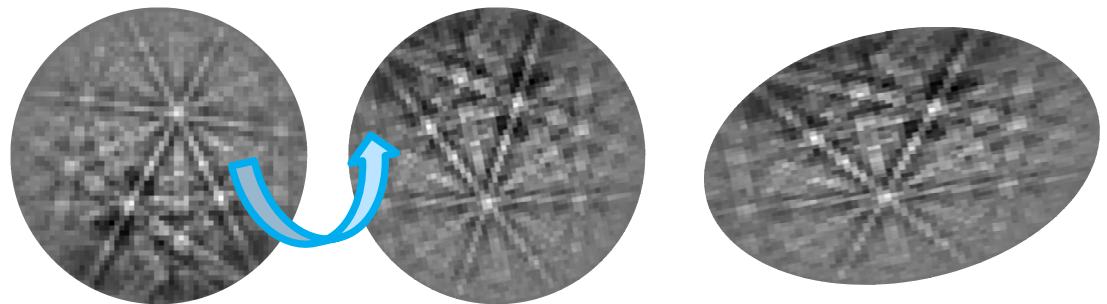
Detector pattern  
(corrected)



Cross-correlation on the sphere



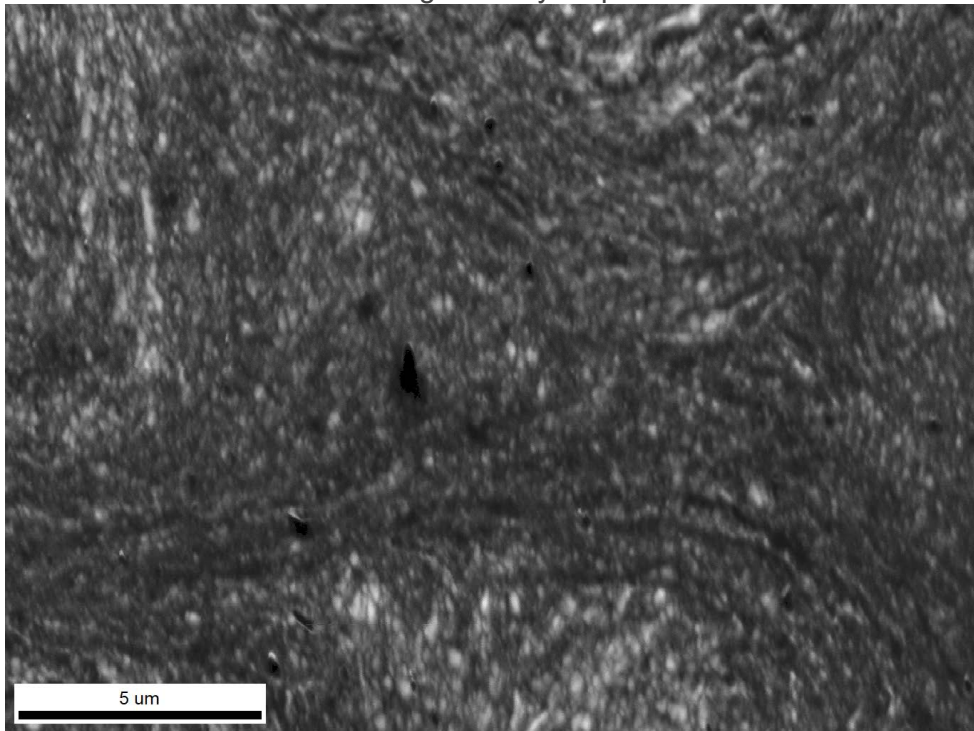
Note: this analog to the 2D lighthouse example is oversimplified. One difference of 3D spherical problem relative to the 2D planar example is the rotational component.



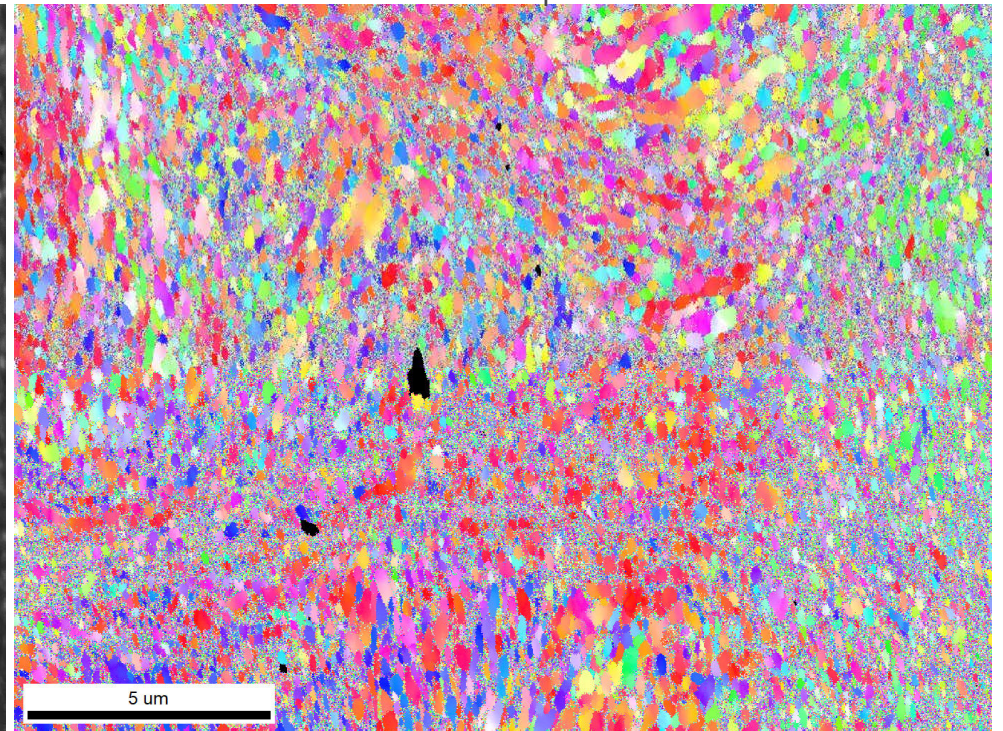
# EBSD mapping using Hough-based indexing

EBSD indexing improvements on highly deformed compressed ferrite powder sample

Image Quality map



IPF map



Hough indexing

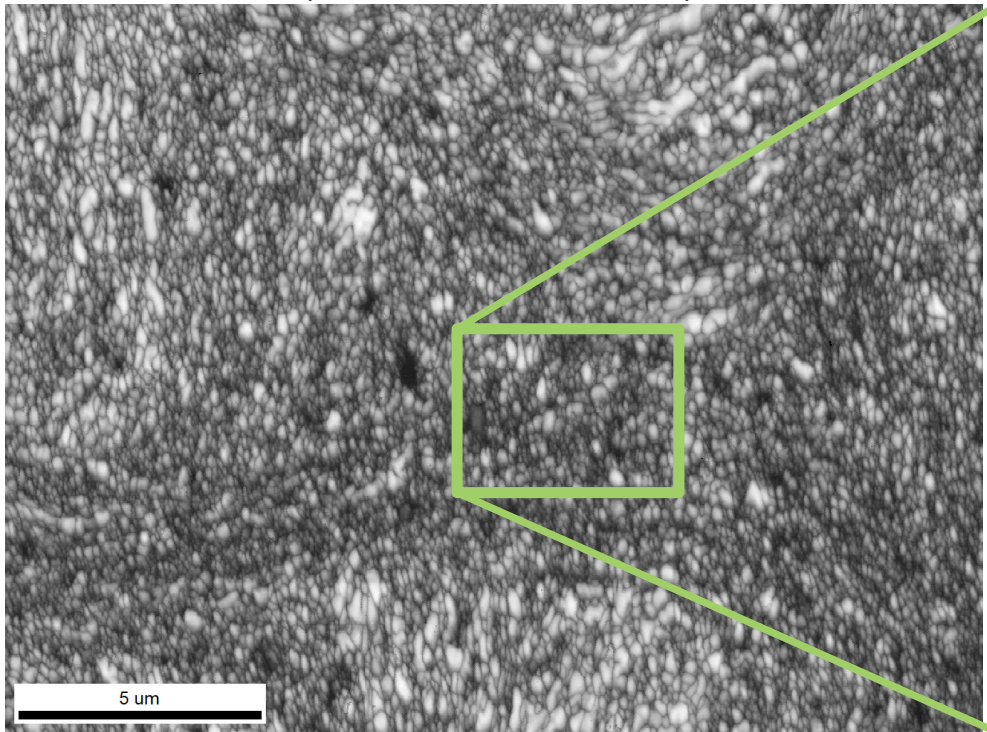
*Sample courtesy of Carsten Bonnekoh, KIT, Germany*



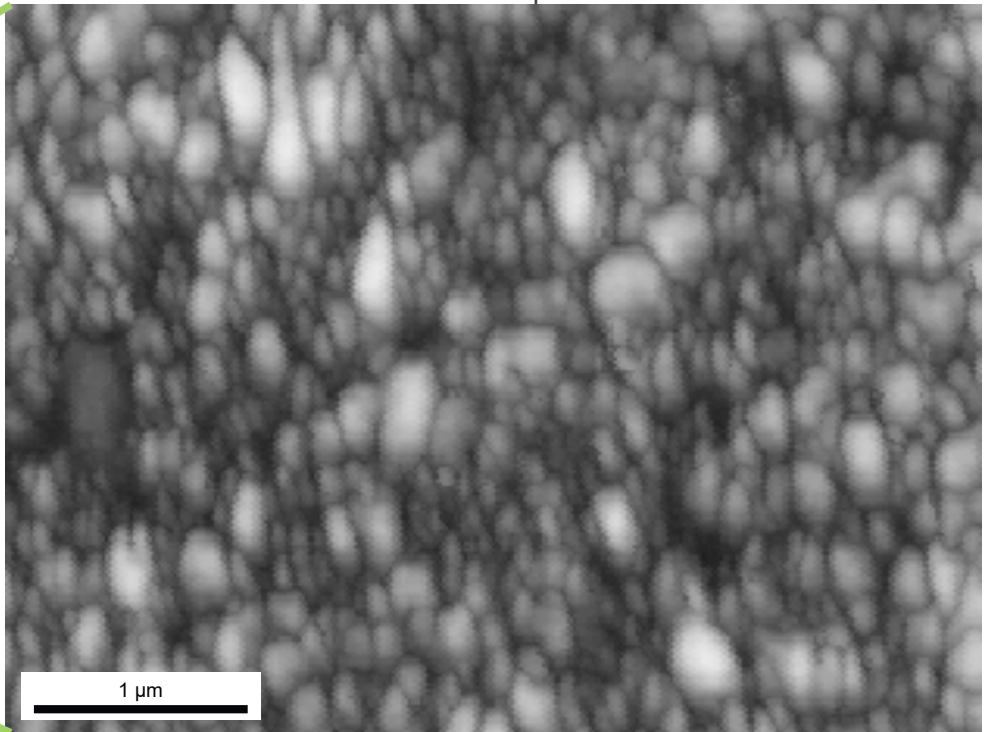
# EBSD mapping using Spherical indexing with NPAR

EBSD indexing improvements on highly deformed compressed ferrite powder sample

Spherical correlation index map



IPF map

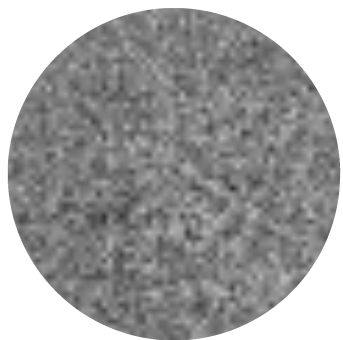


Hough indexing, ISR: 98.1%

*Sample courtesy of Carsten Bonnekoh, KIT, Germany*

# Spherical EBSD indexing robustness

First Pattern



Hough Indexing  
(NPAR)



Dictionary Indexing  
(2.5°)



Bandwidth = 53



Bandwidth = 63



Bandwidth = 74



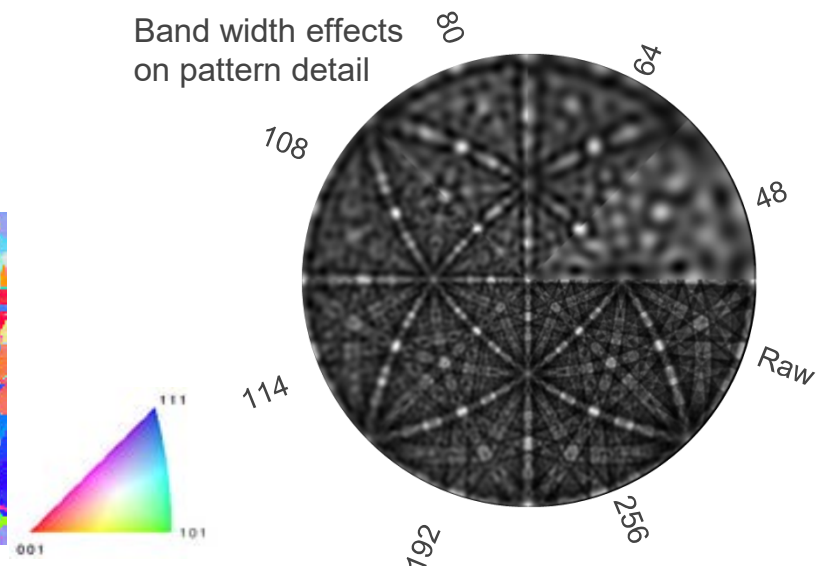
Bandwidth = 88



Bandwidth = 113



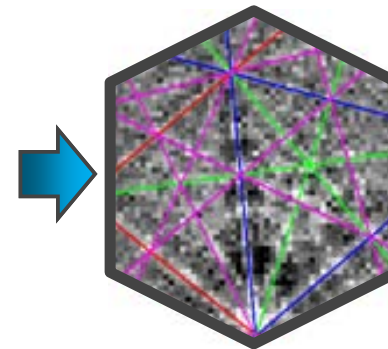
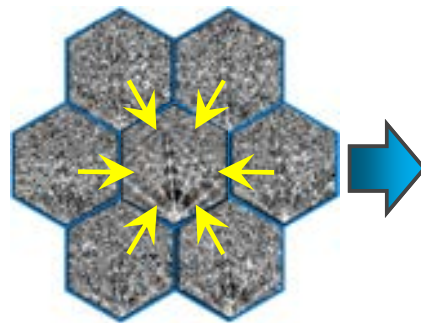
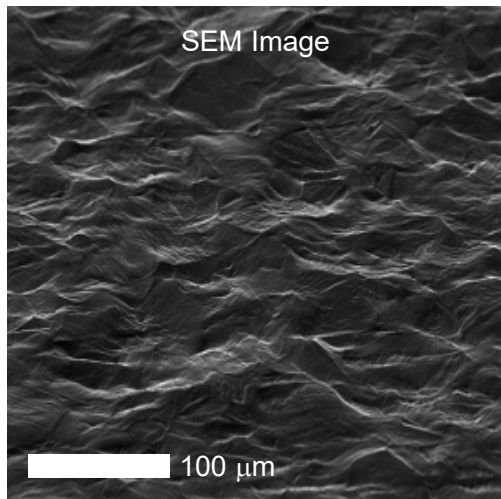
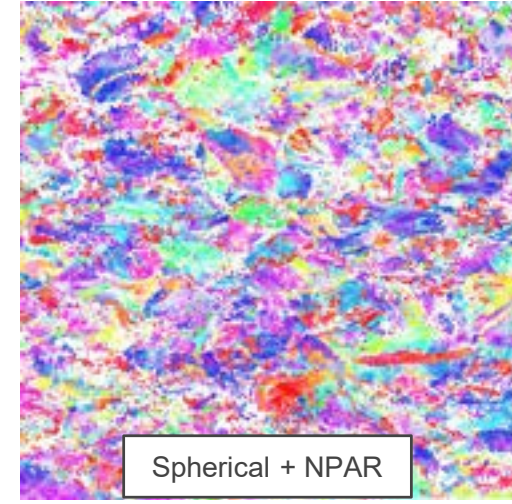
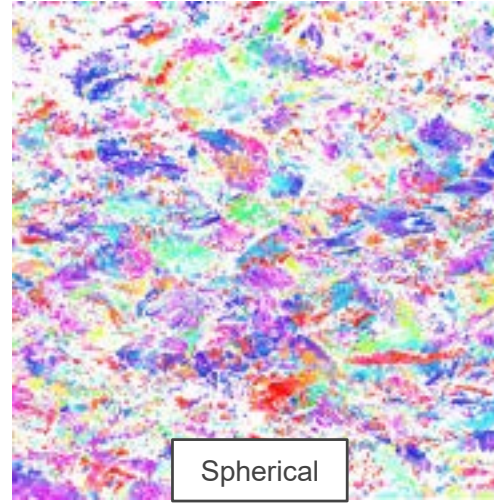
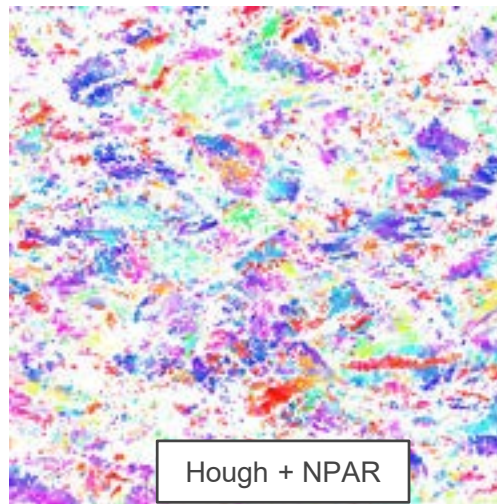
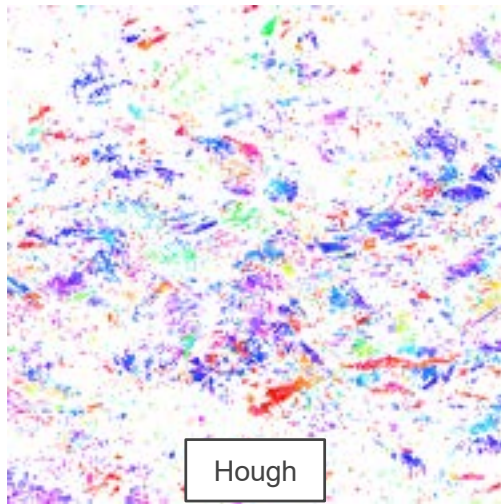
Band width effects  
on pattern detail



Lenthe, W. C., Singh, S., & De Graef, M. (2019). A spherical harmonic transform approach to the indexing of electron back-scattered diffraction patterns. *Ultramicroscopy*, 207, 112841.



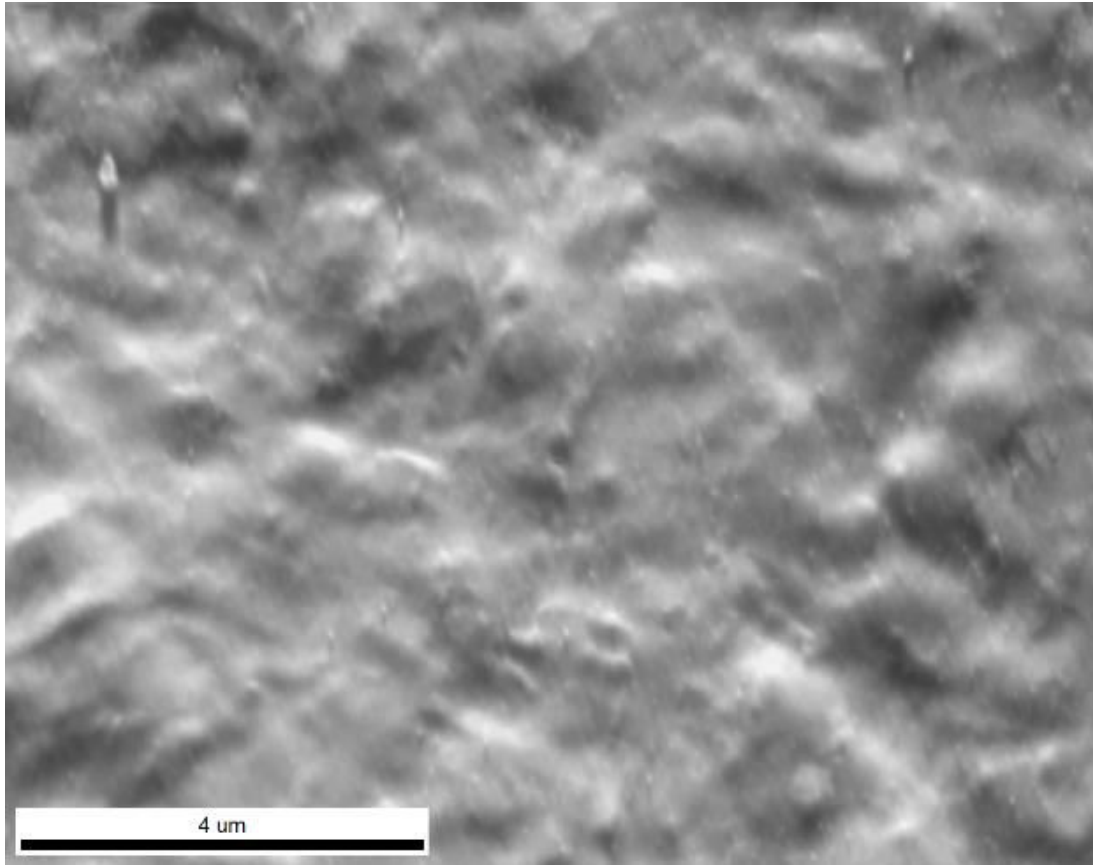
## Tensile sample – rough surface



Brewick, P.T., Wright, S.I. & Rowenhorst D.J. (2019). NLPAR: Non-local smoothing for enhanced EBSD pattern indexing. *Ultramicroscopy*, 200, 50-61.

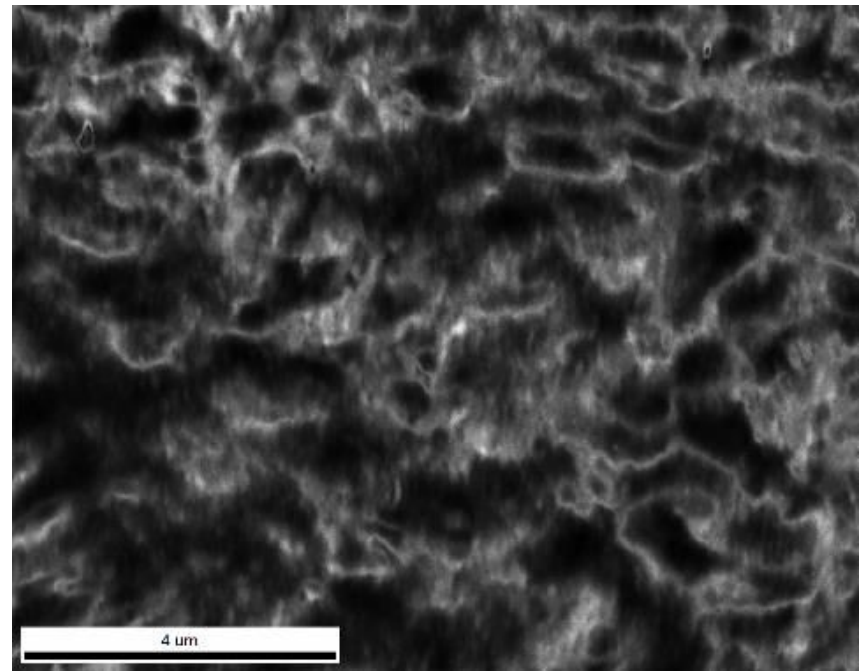


# Titanium nitride coating with sub-micron grain size



SEM image of analysis area

- TiN coating on coarse grained polycrystalline substrate – rough surface
- No additional surface preparation for EBSD

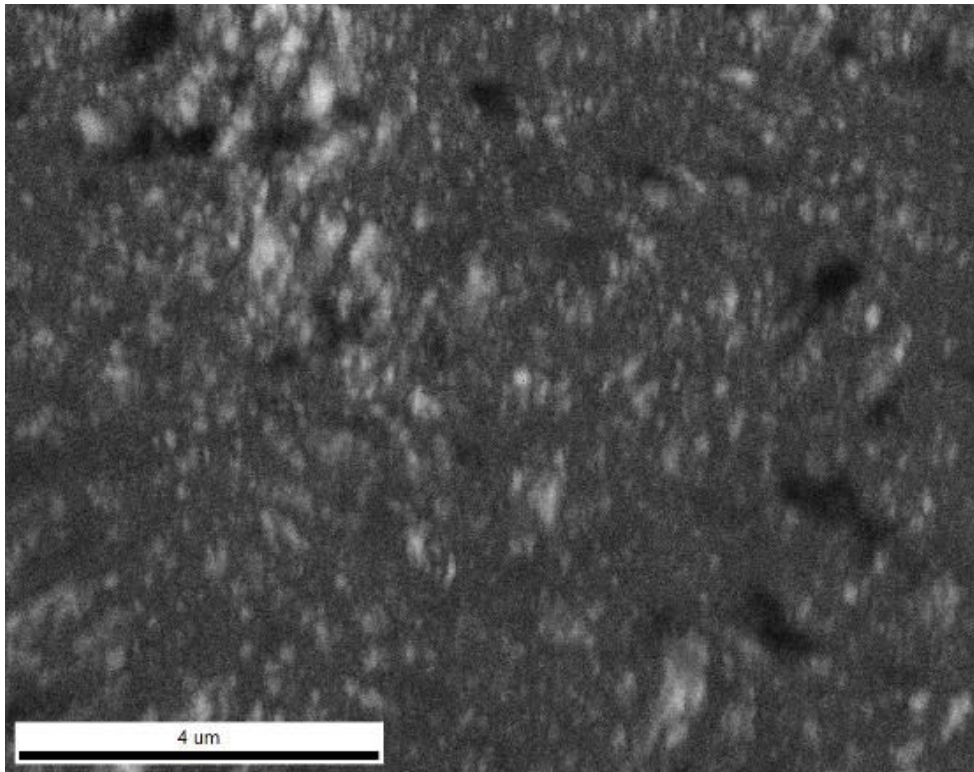


IQ map as measured

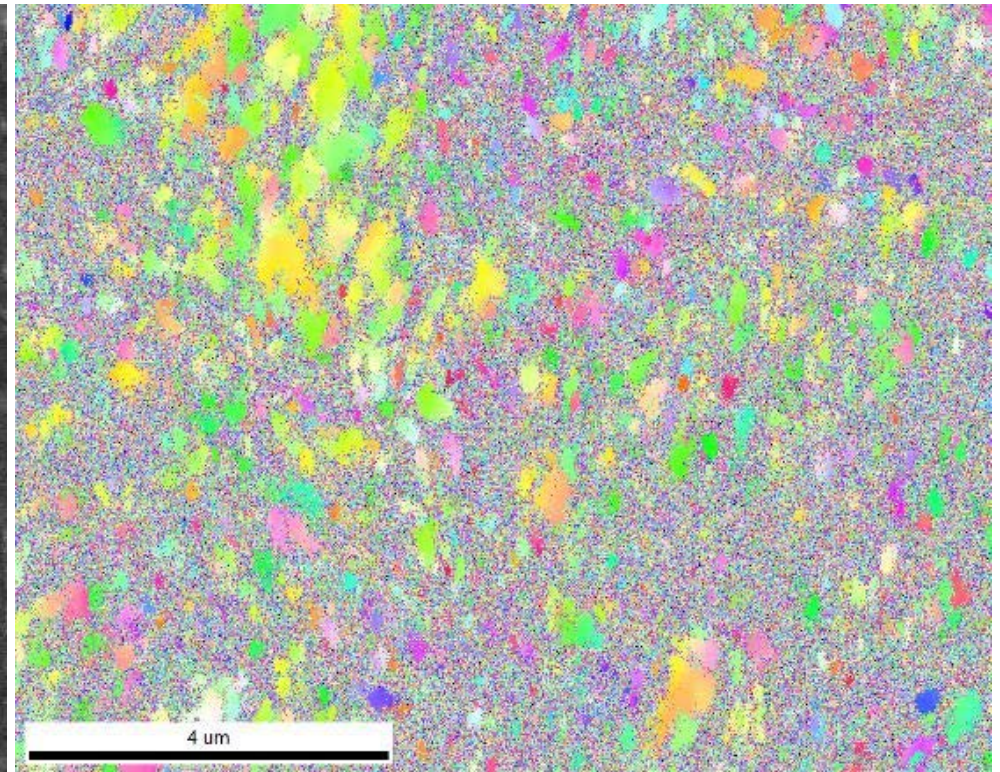
480,217 points, 10.89 x 8.57 μm, 15 nm steps

# Titanium nitride coating – Hough results

- Weak patterns combined with strong overlap at boundaries prevents successful indexing



IQ map after pattern intensity homogenization

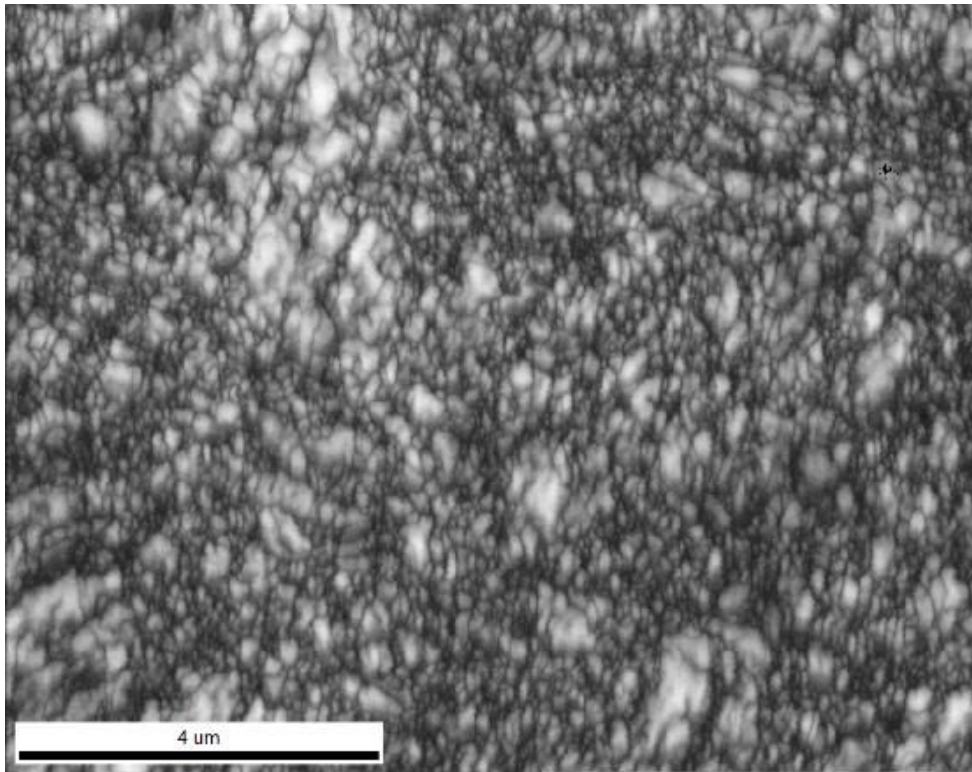


IPF map: Indexing success: 23.2%

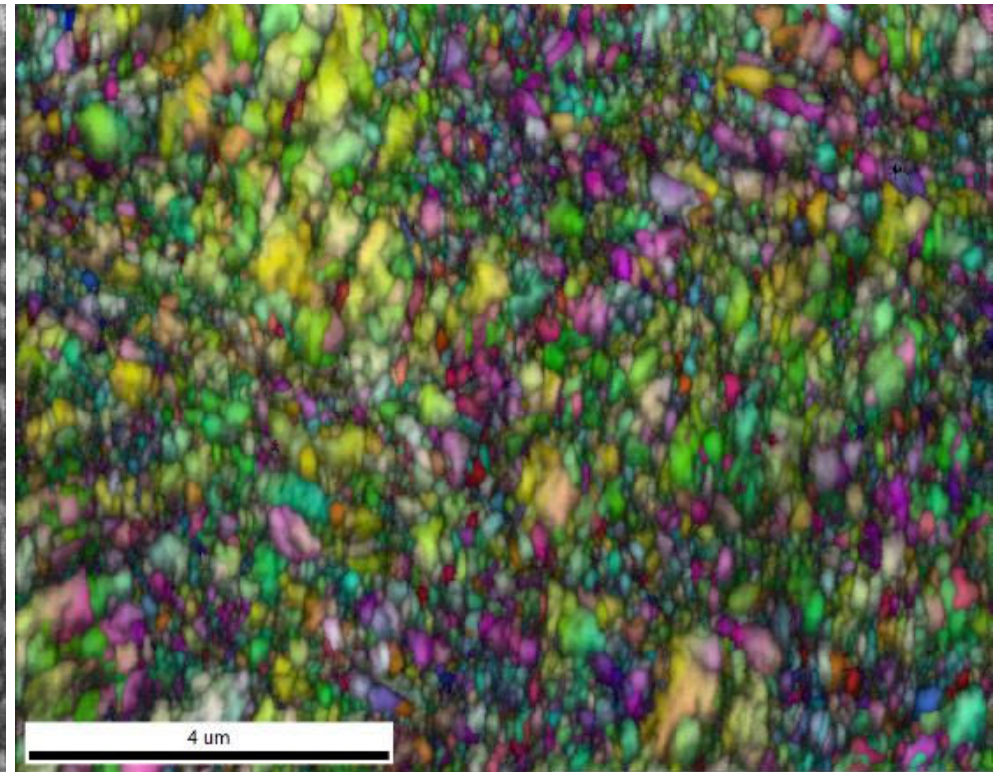


# Titanium nitride coating – Spherical indexing results

- Indexing results with spherical indexing



Spherical correlation map



IPF map – indexing success: 96%



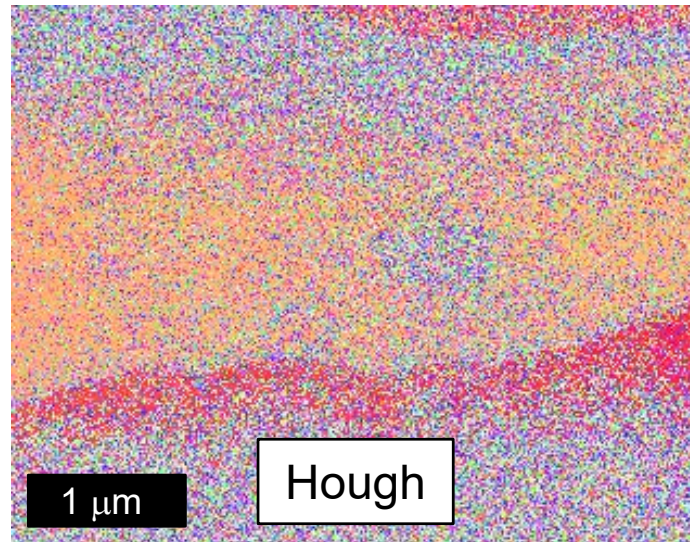
## TKD - ferrite



AHE

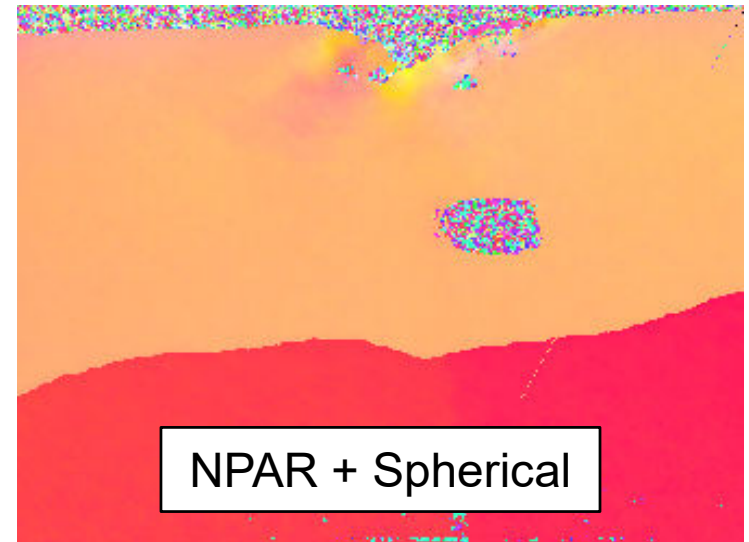


NPAR



1  $\mu\text{m}$

Hough

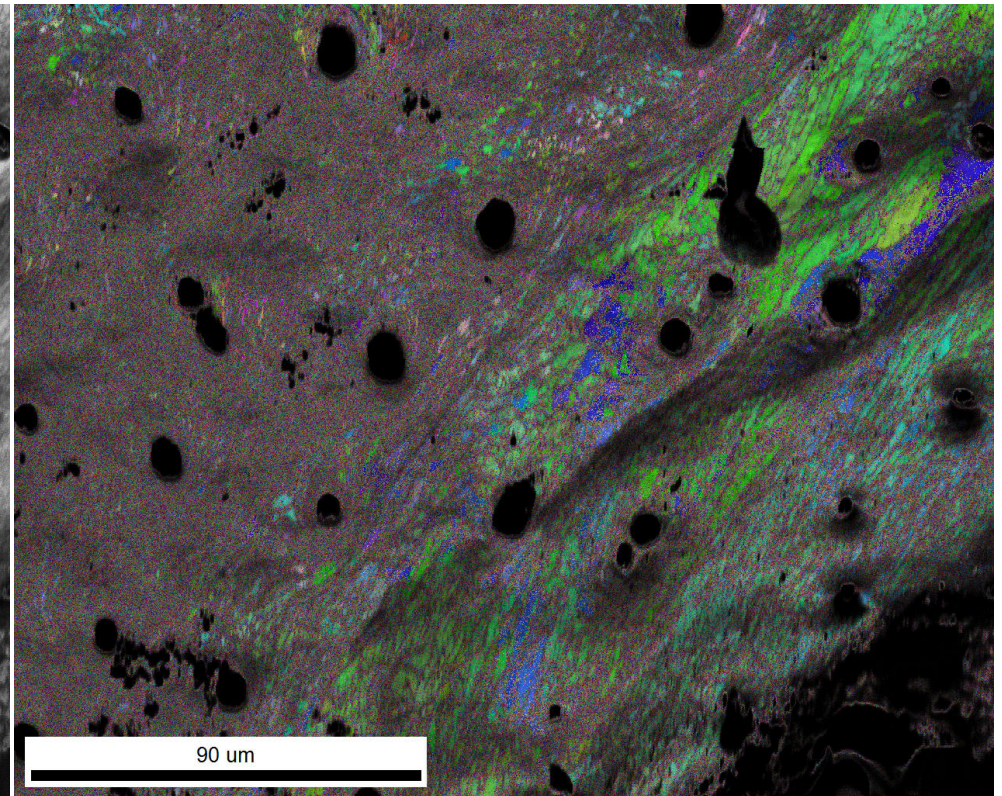
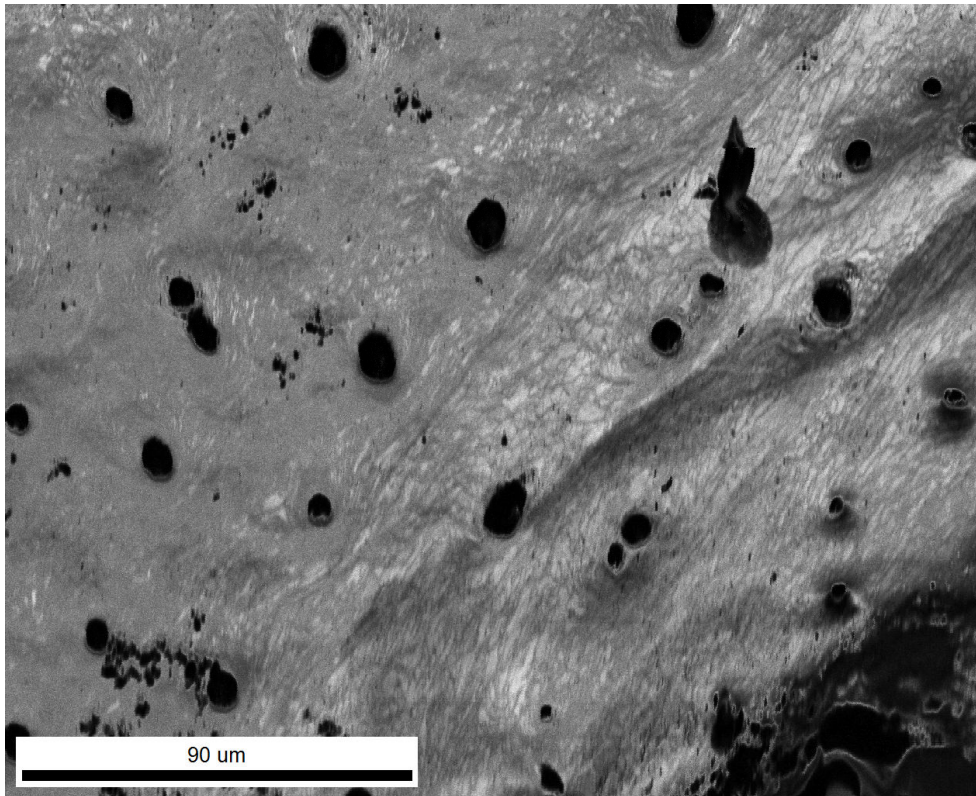


NPAR + Spherical

- TKD samples often show significant gradients in pattern quality and intensity due to thickness variations
  - Image processing using AHE (adaptive histogram equalization) allows consistent matching with the master patterns
- Band width may change strongly within patterns due to the projection geometry
- Spherical indexing can help overcome these challenges



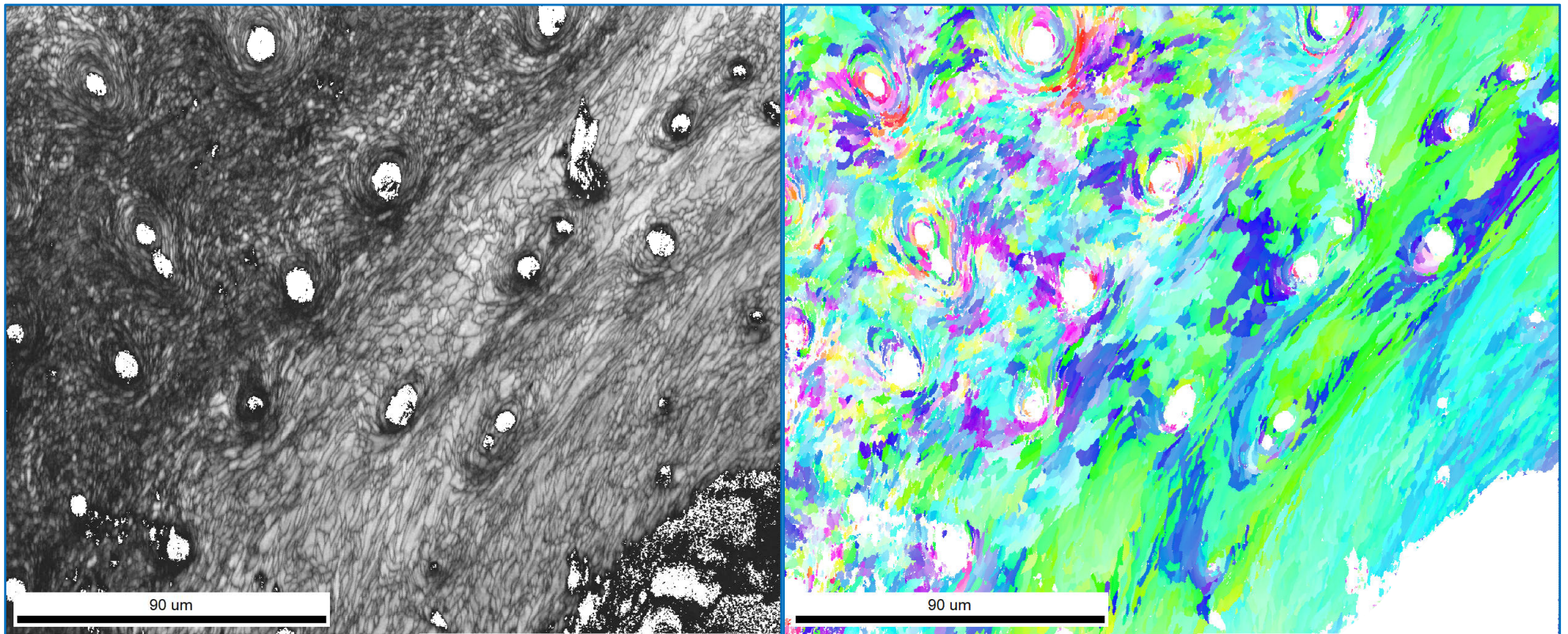
## Beam sensitive bio-mineral example: brachiopod cross-section



Specimen courtesy of E. Griesshaber, Geo- und Umweltwissenschaften, Ludwig-Maximilians-Universität München



## Brachiopod shell – spherical indexing

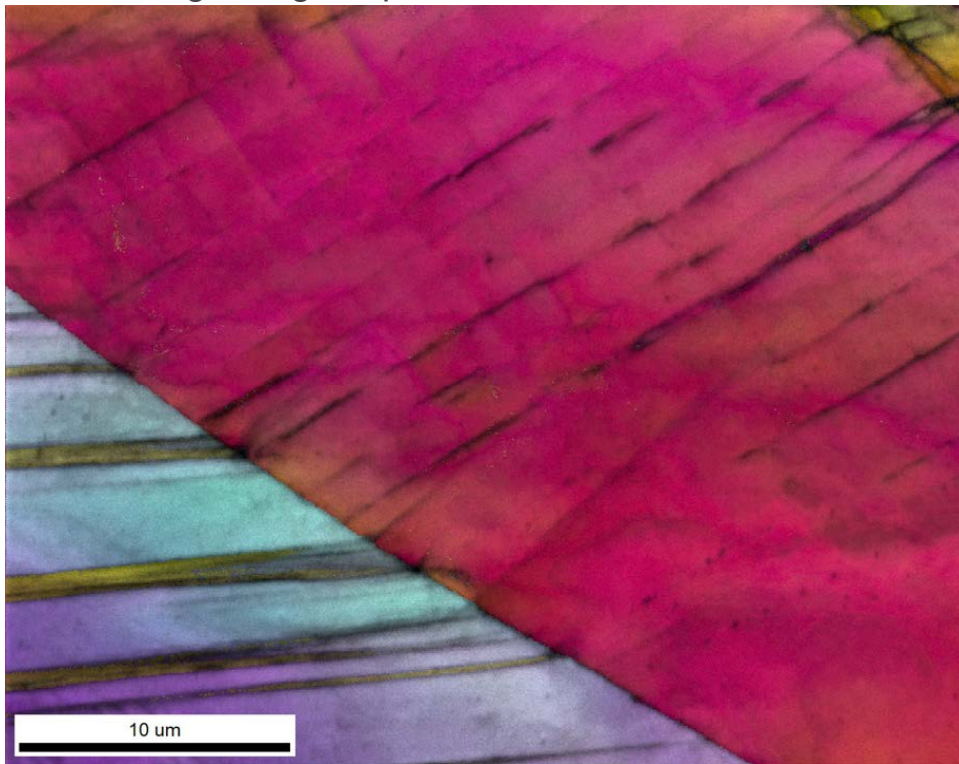


Specimen courtesy of E. Griesshaber, Geo- und Umweltwissenschaften, Ludwig-Maximilians-Universität München

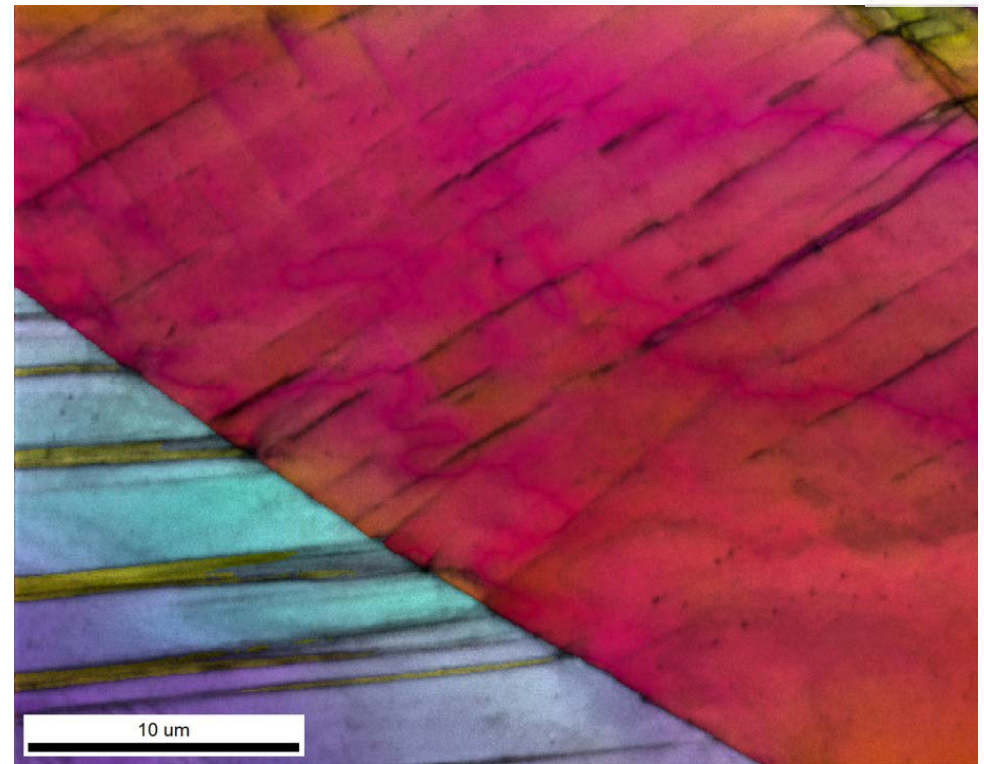


# Spherical indexing – orientation precision improvements

- Because spherical indexing uses the entire pattern, you can get greatly improved orientation precision when working with good patterns



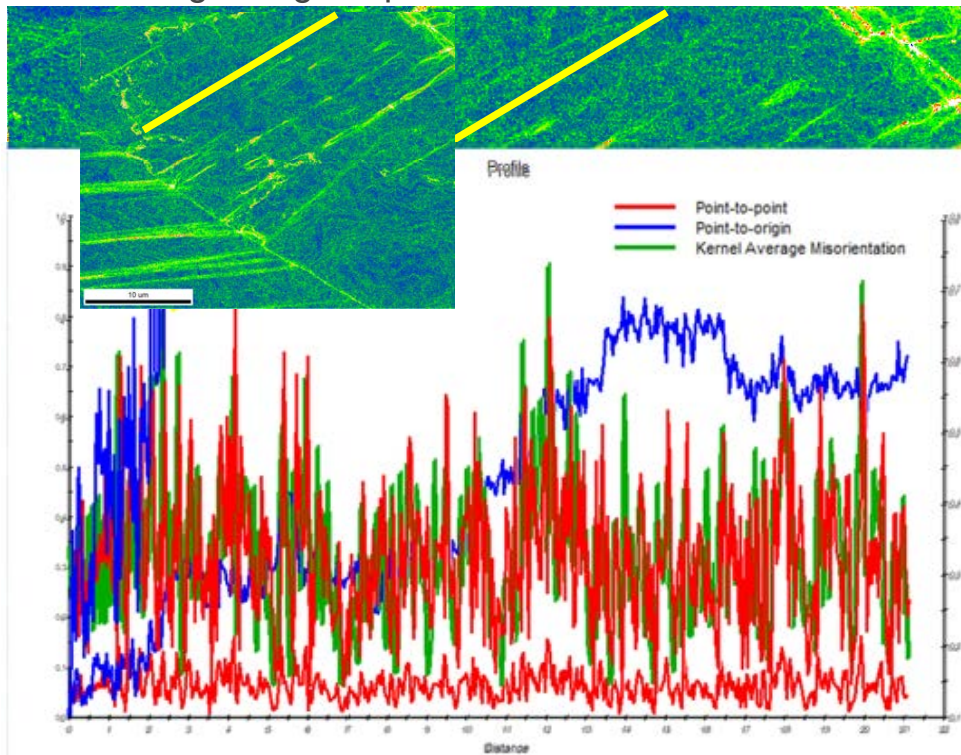
Triplet voting indexing




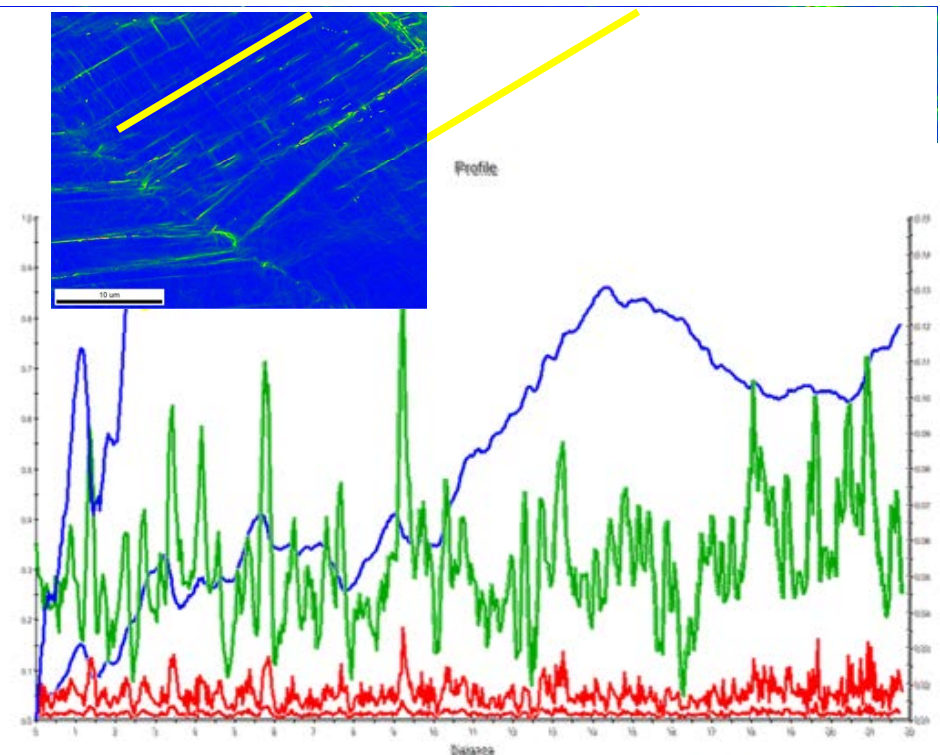
Spherical indexing

# Spherical indexing – orientation precision improvements

- Because spherical indexing uses the entire pattern, you can get greatly improved orientation precision when working with good patterns



Kernel Average Misorientation  0.00 2.50  
Triplet voting indexing



Precision better than  $0.05^\circ$

Spherical indexing

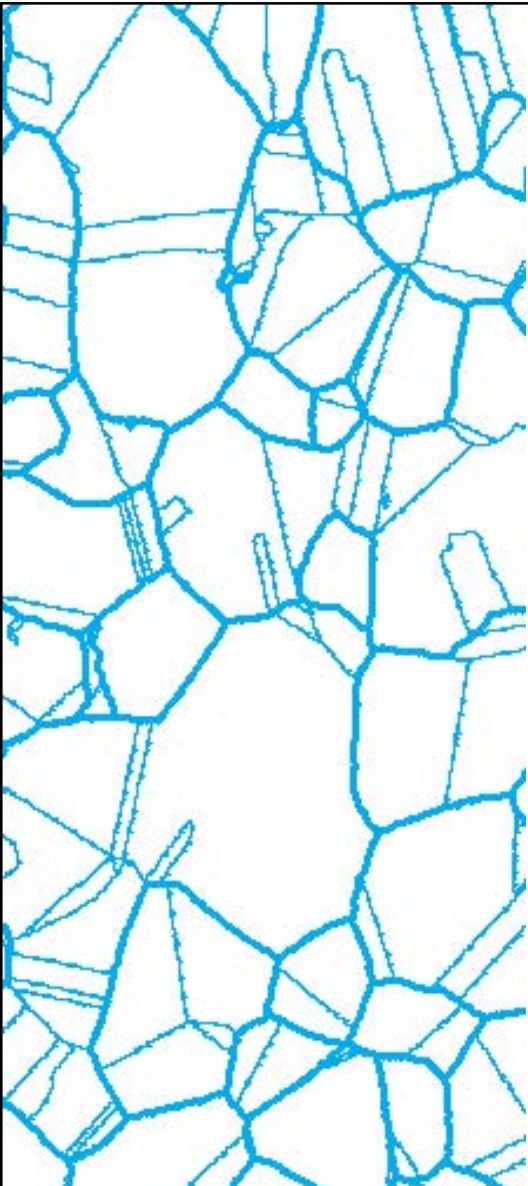
# Summary

## Spherical Indexing

- + As robust as Dictionary Indexing but faster
- + Higher angular precision than Hough
- + No pre-calculation of a dictionary required
- Slower than Hough
- Master Pattern required
- Image Processing: Adaptive Histogram Equalization

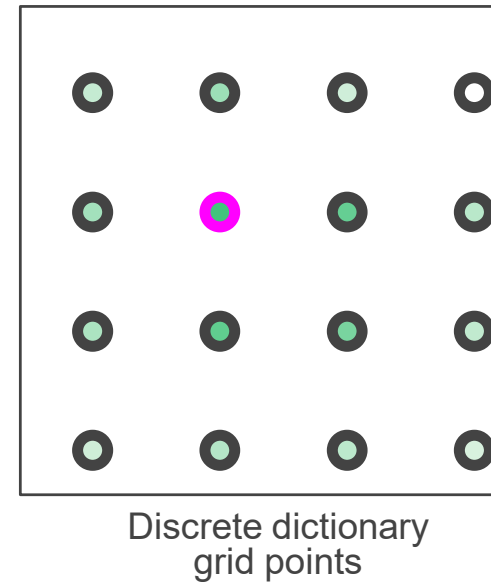
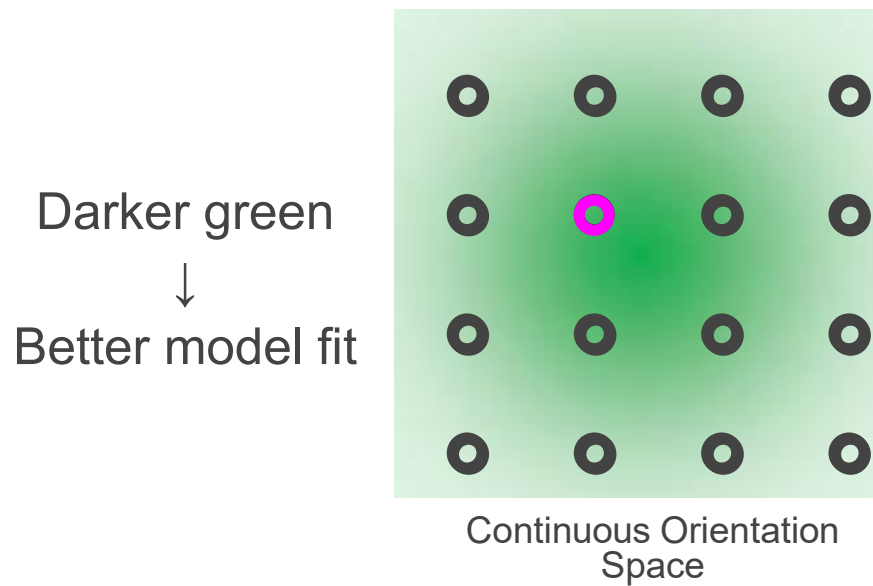
**The robustness of spherical indexing doesn't mean you no longer have to prepare your samples well!!!**





## Orientation refinement

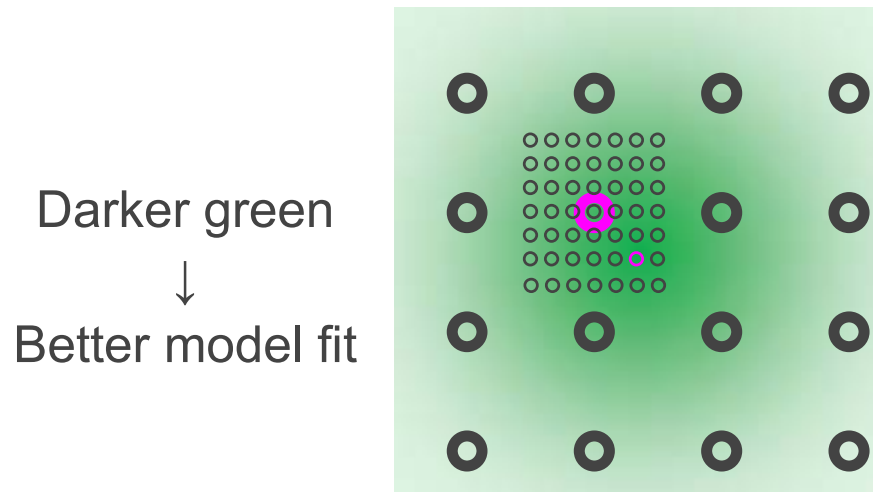
## Orientation refinement: 2D example



## Refinement for increased angular precision

- No good match is found for too large of step size
- Too small of orientation step sizes are intractable
- Problem becomes intractable before desired precision

## Orientation refinement: 2D example



## Realspace refinement

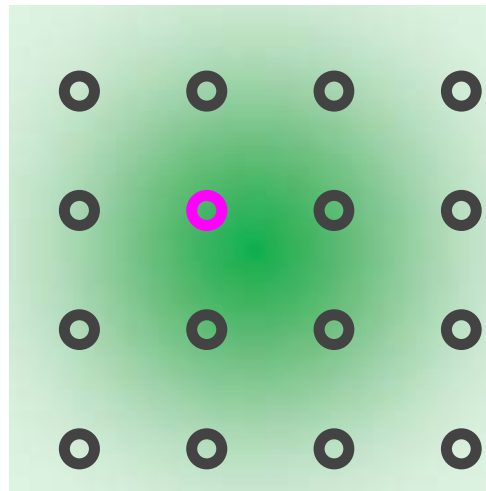
- No good match is found for too large of step size
- Too small of orientation step sizes are intractable
- Problem becomes intractable before desired precision
- Re-index using a high precision localized dictionary (in 3D)



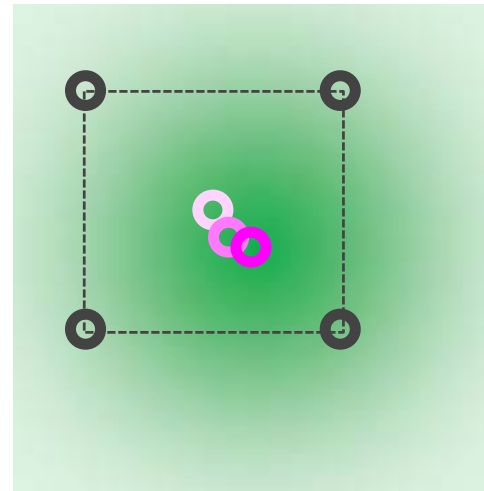
# Orientation refinement: non-linear optimization example

## Bound Optimization By Quadratic Approximation

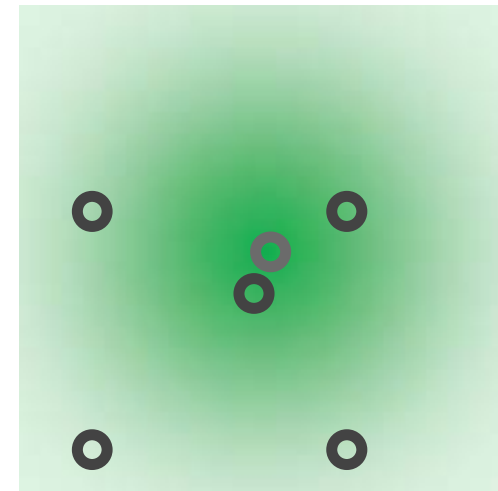
- Bounds are based on indexing grid
- Optimize to maximize normalized dot product
  - At each iteration choose a new iteration point by:
  - Fitting a Quadratic Approximation to the current points
  - Selecting the maxima of the quadratic constrained by the bounds



Initial bounding dot products



Estimate best point



Iterate until convergence

- Non-linear iterative optimization to refine orientation
- Bounded by specified dictionary parameters
- We are using a variant on the BOBYQA approach more suitable to EBSD

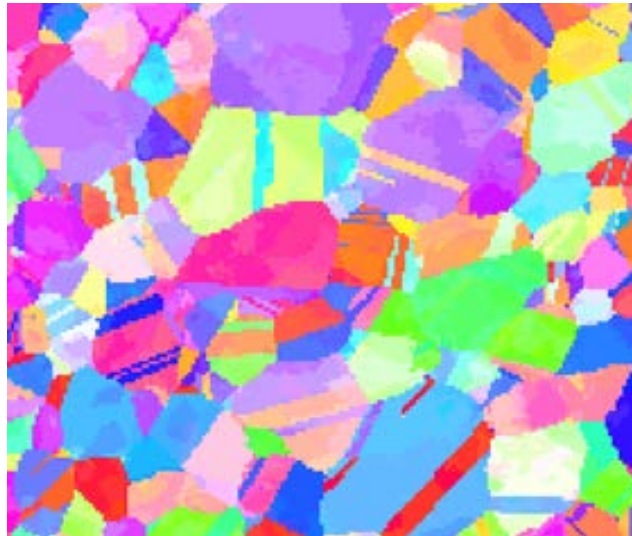
# Realspace refinement

Hough Based



Continuous orientations

2.5° Dictionary



Discretized indexable orientations  
before refinement

Refined

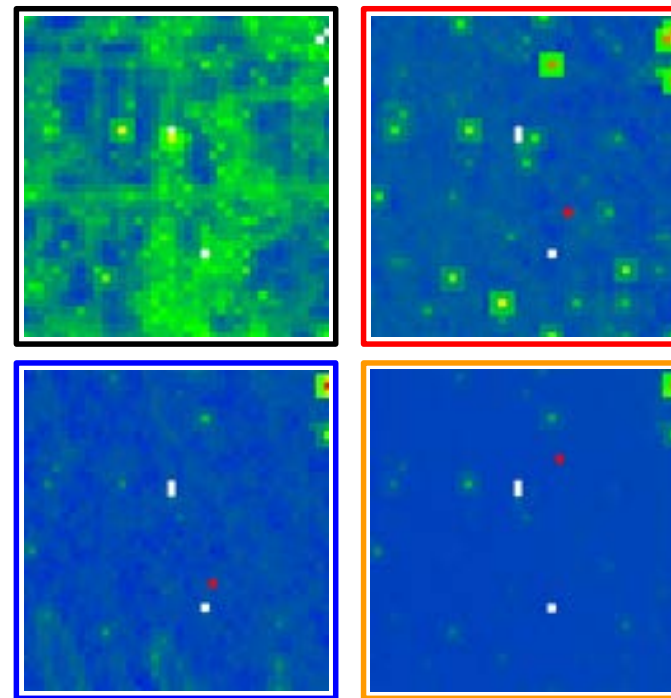
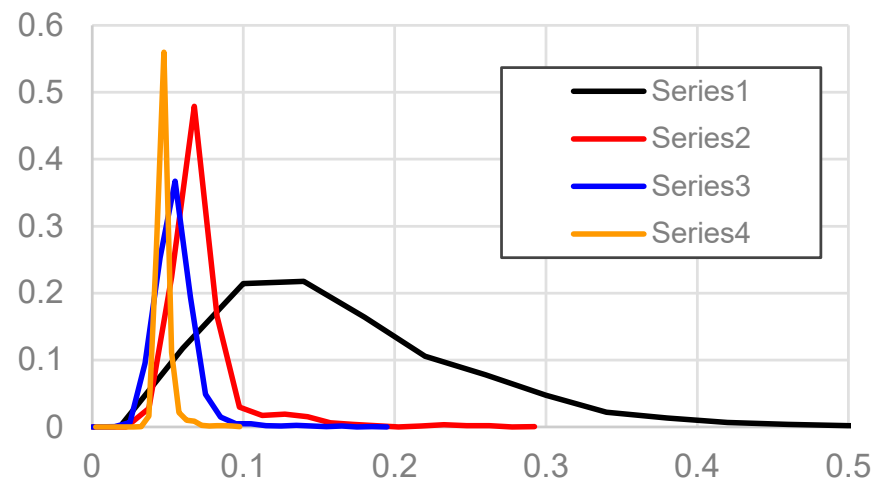
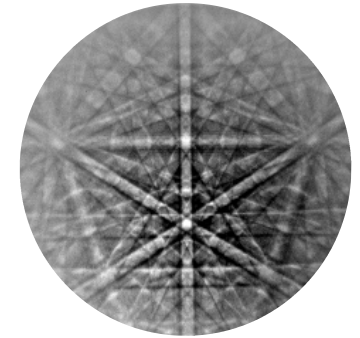
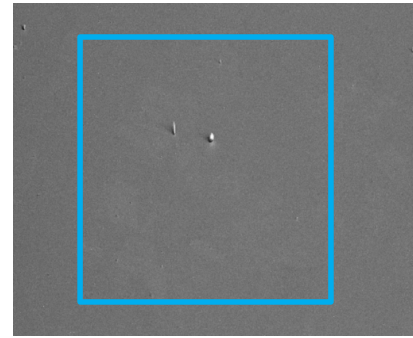


Continuous orientations after refinement

Increasing Bandwidth for Spherical Indexing

# Preliminary precision tests

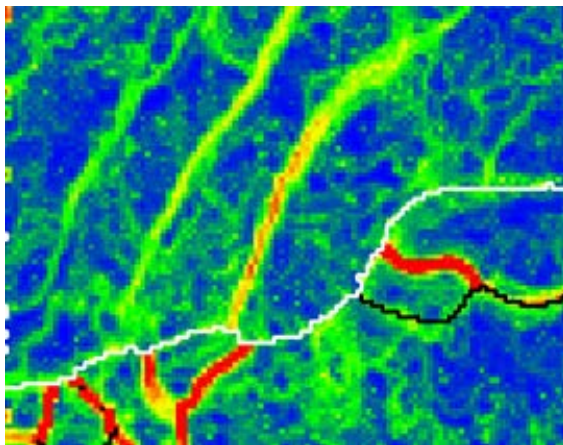
- Single crystal silicon
- 1 mm x 1 mm scan
- 480 x 480-pixel patterns
- 25-micron step size



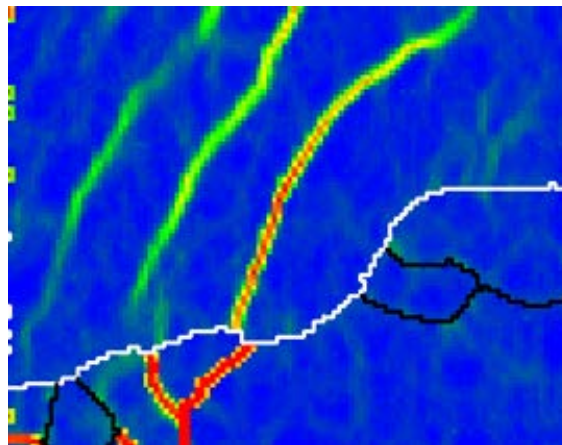


# Cross-correlation comparison

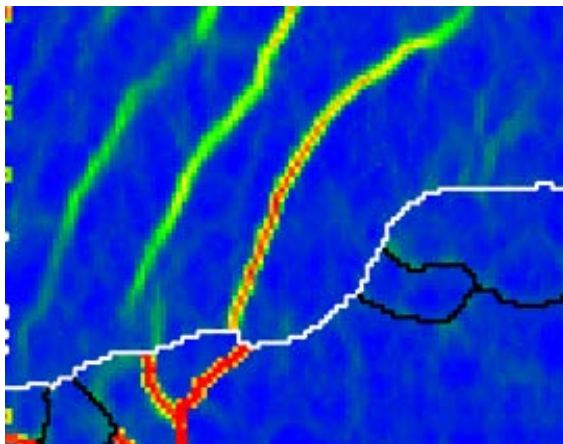
Hough



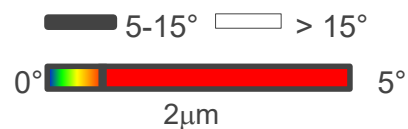
Hough + Refinement



Spherical + Refinement

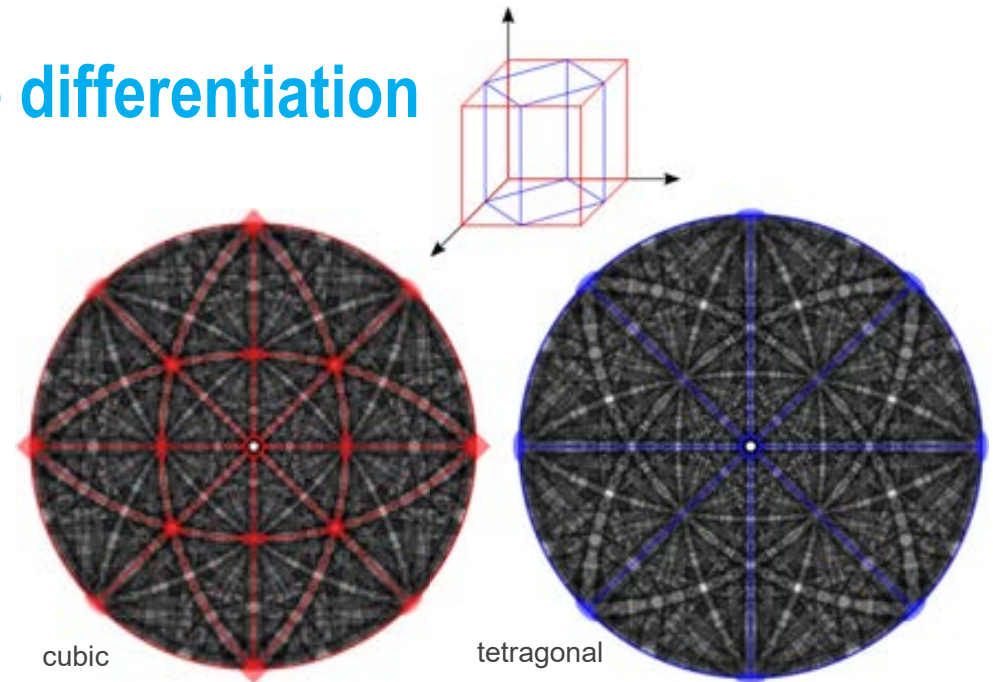


OpenXY  
Courtesy Josh Kacher  
Georgia Tech University



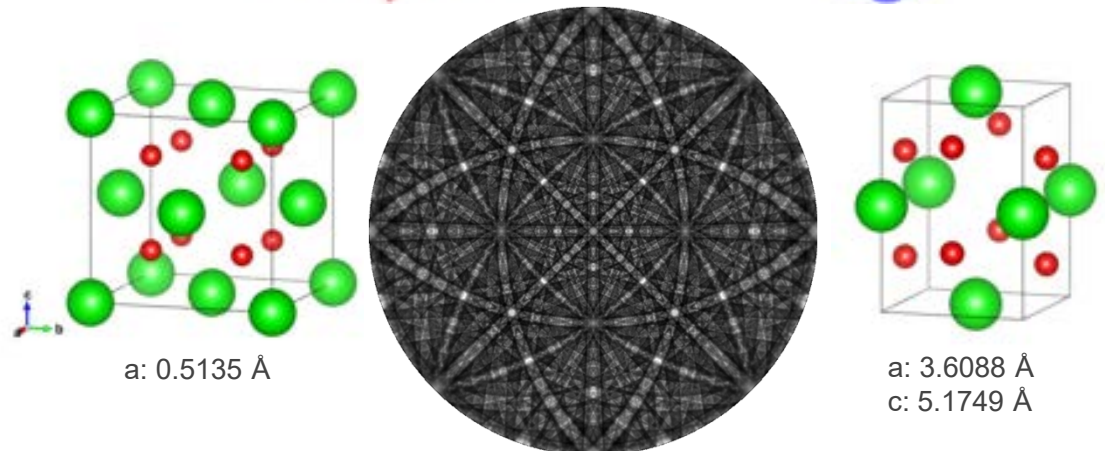
# ZrO<sub>2</sub> pseudosymmetry & phase differentiation

- Tetragonal phase is small strain away from cubic
- Double cell c/a ratio 1.0134
- ~0.7% strain between phases
- Type and severity of pseudo symmetry is predicted well w/ spherical cross correlation



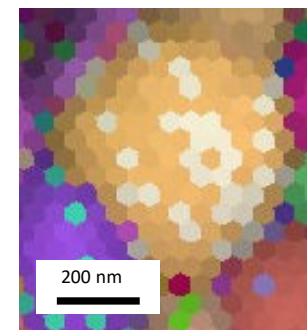
Normalized cross-correlation between the master patterns for each phase for all rotations

Max Cross Correlation	Cubic	Tetragonal
Cubic	60@<111> 29.9%	45@<001> 96.3%
Tetragonal		90@<110> 90.5%

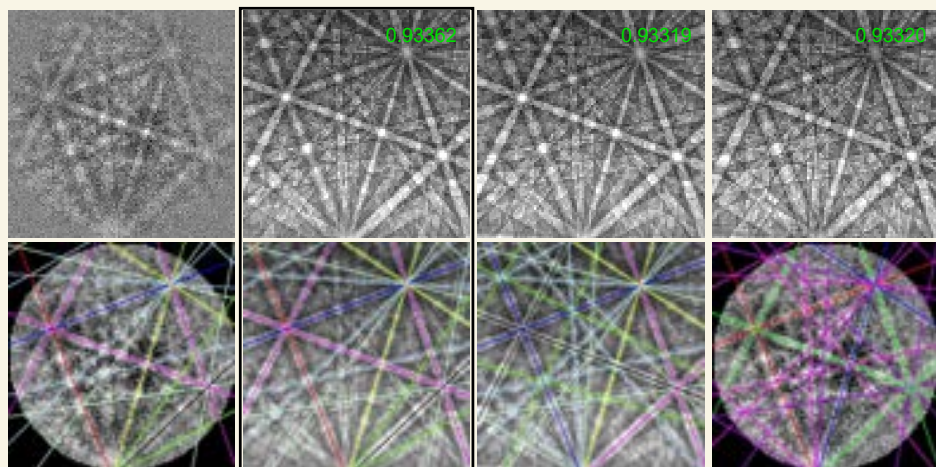


Cubic vs tetragonal rotated 45 @ z

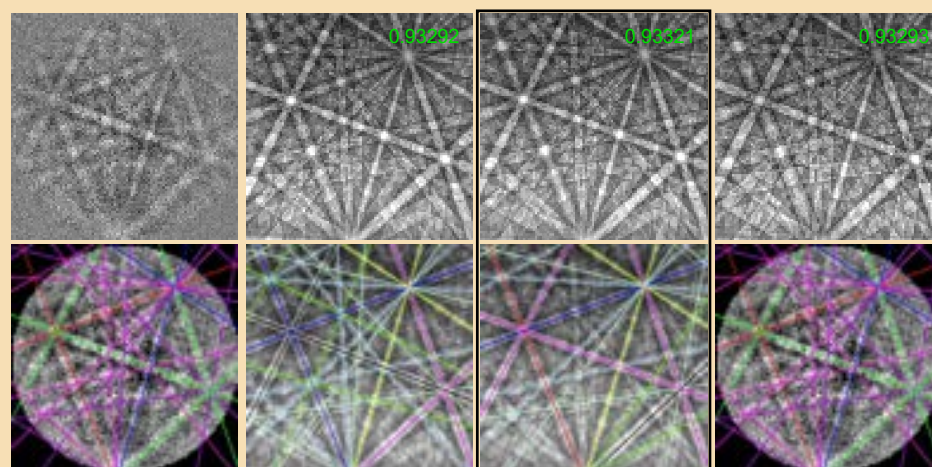
# ZrO<sub>2</sub> pseudosymmetry



Experimental Pattern      Tetragonal Orientation      Tetragonal Pseudosymmetric Orientation      Cubic Orientation

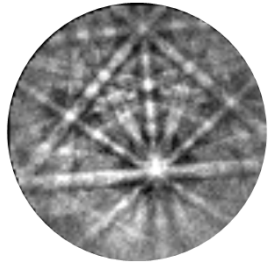


Experimental Pattern      Tetragonal Orientation      Tetragonal Pseudosymmetric Orientation      Cubic Orientation



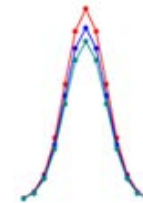


# Pseudosymmetry / phase differentiation workflow



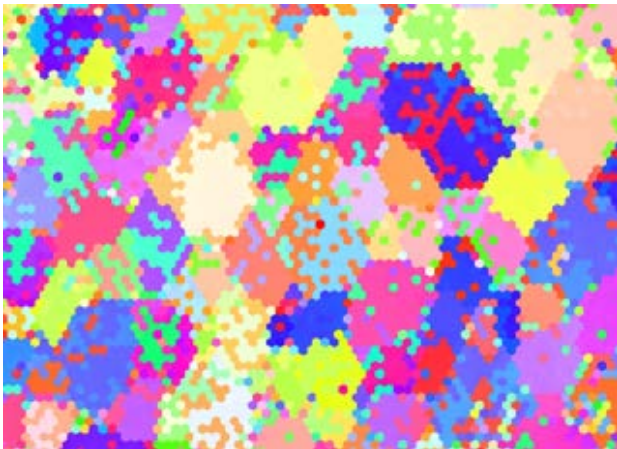
## Experimental Pattern

- Find the orientation with the highest normalized dot product (NDP) for the cubic phase ( $q_c$ )
  - Refine the cubic orientation ( $q_c^R$ )
- Find the orientation with the highest NDP for the tetragonal phase -or-
  - Calculate the tetragonal orientation related to the cubic orientation ( $q_t$ )
    - Refine the tetragonal orientation ( $q_t^R$ )
- Calculate the pseudosymmetric variation of  $q_t$  ( $q_{t2}$ )
  - Refine  $q_{t2}$  ( $q_{t2}^R$ )
- Select refined orientation with the maximum NDP



# ZrO<sub>2</sub> tetragonal pseudosymmetry results

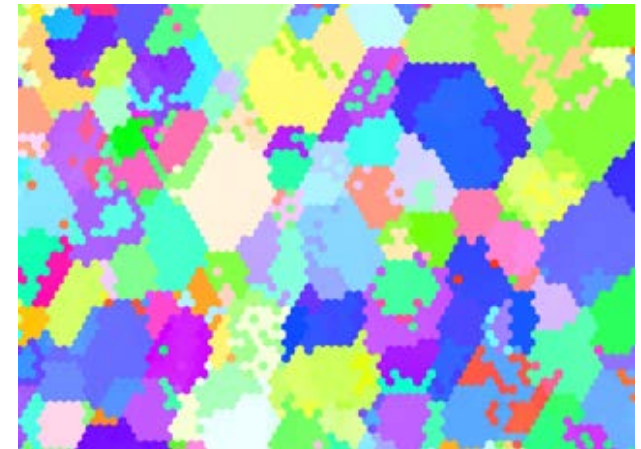
Hough



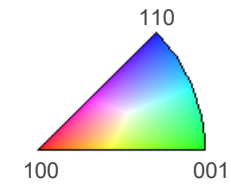
Spherical



Spherical + Pseudosymmetry  
Analysis



1 μm

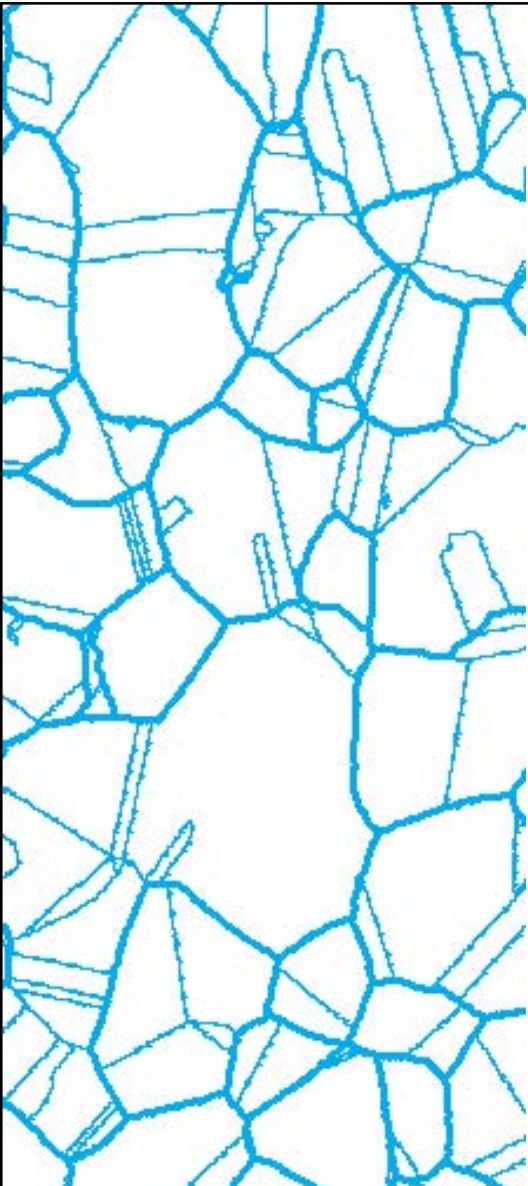


# Summary – orientation refinement

## Refinement (OIM Matrix)

- Can start with Hough, DI, or SI results
- Improved angular resolution
  - Improved GNDs and KAMs
  - Rivaling HR-EBSD orientation precision (but no lattice distortion measurement)
- Master pattern and recorded patterns required
- Yet to explore:
  - Effects of pattern size and quality
  - Effects of lattice distortion (HR-EBSD)
  - ...





Thank you for your attention